

Estimating the strength of rock materials

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SYNOPSIS

A practical approach for estimating the strength behaviour of rock materials is presented. The demand for data on rock properties in engineering design is considered, and it is shown that, based on the trends observed in the South African mining industry during the last twenty years, simple and easy-to-use methods for the estimation of the uniaxial and triaxial strength of rock materials are needed.

It is shown that the uniaxial compressive strength of rock can be conveniently determined from the point-load strength index, which is obtained underground on unprepared rock cores.

The triaxial strength of rock can be estimated for most practical purposes from empirical strength criteria. Two such criteria are proposed that allow estimation of the triaxial strength for rock materials to about 10 per cent. The only input required for these criteria is the uniaxial compressive strength.

The validity of the practical approach outlined in this paper is confirmed from the experimental tests conducted on some 700 rock specimens representing five rock types. A practical example for the prediction of rock strength is given.

SAMEVATTING

'n Praktiese benadering van die raming van die sterktegedrag van rotsmateriale word uiteengesit. Die vraag na gegewens oor rotseienskappe in ingenieursontwerp word oorweeg en daar word aan die hand van die neigings wat gedurende die afgelope twintig jaar in die Suid-Afrikaanse mynboubedryf waargeneem is, getoon dat daar 'n behoefte bestaan aan eenvoudige en maklike metodes om die eenassige en drie-assige sterkte van rotsmateriaal te bepaal.

Daar word getoon dat die eenassige druksterkte van rots maklik bepaal kan word aan die hand van die punt-las sterkte-indeks wat ondergronds van onbereide rotskerns verkry word.

Die drie-assige sterkte van rots kan vir die meeste praktiese doeleindes op grond van empiriese sterktekriteria geraam word. Daar word twee sulke kriteria aan die hand gedoen wat die raming van die drie-assige sterkte van rotsmateriale tot ongeveer 10 persent moontlik maak. Die enigste waarde wat vir hierdie kriteria nodig is, is die eenassige druksterkte.

Die geldigheid van die praktiese benadering wat in hierdie verhandeling uiteengesit word, word bevestig deur die eksperimentele toetse wat uitgevoer is op ongeveer 700 rotsmonsters wat vyf soorte rots verteenwoordig. Daar word 'n praktiese voorbeeld vir die voorspelling van rotssterktes gegee.

INTRODUCTION

The design of excavations in rock involves, among other things, the evaluation of the mechanical properties of rock materials. For reasons of economics, it is never feasible to measure fully the characteristics of a complex rock mass. This is obvious from examinations of Table I, in which the many factors influencing rock behaviour are listed. Consequently, the ability to predict the strength behaviour of rock materials is very important for the design engineer because it allows him better appreciation of the problem in hand, clarifies the influence of different variables, and provides an estimate of the design parameters. While it is recognized that a rock material is only a part of a complex rock mass, the strength behaviour of a rock material is nevertheless significant. After all, a sample of rock material sometimes represents a small-scale model of the rock mass since they have both gone through the same geological cycle. Furthermore, the characteristics of a rock mass should

not be separated from the characteristics of the rock materials, and, in fact, the interdependence goes so far that a hypothesis derived for rock materials was found¹ to be applicable to rock masses *in situ*. For deep-

level gold mining in South Africa, Ortlepp and Cook² found that mechanical properties determined in the laboratory from small specimens provide a useful description of the characteristics of the strata. They

TABLE I
FACTORS INFLUENCING BEHAVIOUR OF ROCK

I. CHARACTERISTICS OF ROCK	
(i)	<i>Rock material structure</i> Lithology, anisotropy, cracks, and pores
(ii)	<i>Rock mass structure</i> Discontinuities (joints, faults, bedding planes, etc.), their type, orientation, continuity, roughness, waviness, spacing, and length
(iii)	<i>Properties</i> Mechanical, physical, and chemical properties of rock material, discontinuities, and rock mass
II. SPECIMEN AND ENVIRONMENTAL CONDITIONS	
(i)	Moisture content, temperature, and pore-pressure conditions
(ii)	Ground-water conditions and chemical environment (weathering)
(iii)	Specimen size and shape
III. STATE OF STRESS OR STRAIN	
(i)	Magnitude of applied stress or strain
(ii)	Distribution of stress or strain (uniformly and non-uniformly distributed tension, compression, bending or torsion; effects of specimen ends, platens, and machine)
IV. METHOD OF LOADING	
(i)	Type of loading (uniaxial, biaxial, triaxial, tensile, compressive, or shear components)
(ii)	Rate of loading: slow (static) loading: lower than 10 MPa/s rapid (dynamic) loading: 10 to 10 ⁶ MPa/s instantaneous (impact) loading: > 10 ⁶ MPa/s
(iii)	Pattern of loading (constant load, gradually increasing monotonically, repetitive (fatigue), pulse, or alternating)

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TABLE II

SUMMARY OF THE CSIR ROCK TESTING SERVICES* TO THE MINING INDUSTRY DURING PAST 20 YEARS

Property requested	Requests since 1.12.1952	Rock samples tested	Specimens tested	Specimens per rock sample
Uniaxial compressive strength	72	2277	6597	2,89
Uniaxial tensile strength . . .	12	249	316	1,27
Triaxial strength	13	261†	964	3,69
Modulus of elasticity †	54	607	3072	5,06
Poisson's ratio ‡	53	565	2882	5,10
TOTAL (first three properties)	97	2787	7877	2,83

NOTES: *Excluding research tests
 †Includes different stress ratio conditions
 ‡Determined on the same specimens as for strength properties

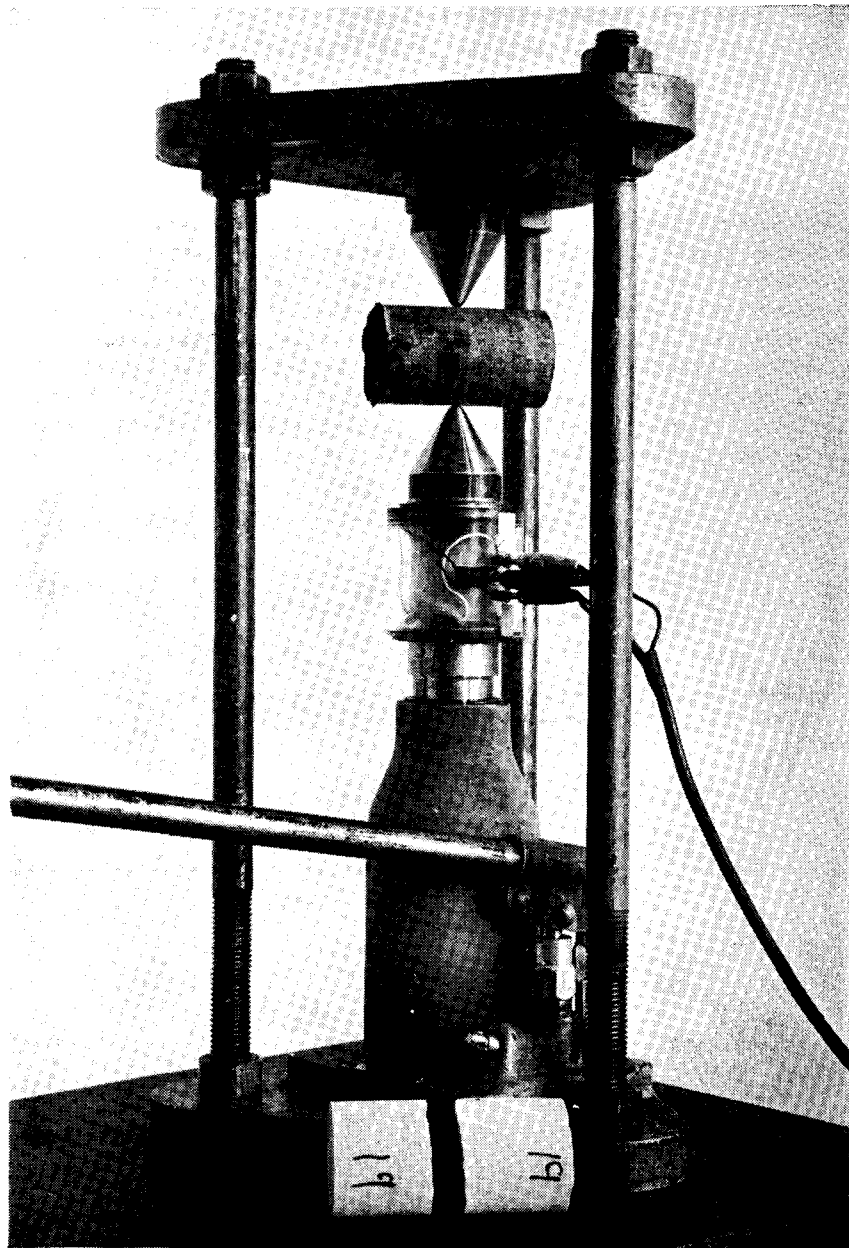


Plate I—Simple equipment for the point-load strength test. Its loading capacity is 125 kN and it costs R150 to manufacture.

also concluded that it might be realistic to use laboratory-determined fracture criteria for the prediction of failure in the underground situation.

In view of these arguments, one should examine what rock-material properties are needed for practical engineering design.

USE OF ROCK PROPERTIES IN DESIGN

What rock properties are required by the design engineer? How are they obtained? Must the results be accurate? To answer these questions, a study was made of the requests for rock testing submitted to the Rock Mechanics Division of the CSIR over a period of twenty years. A clear *modus operandi* in the past and the current needs for the property testing of rock material have emerged. Table II provides some information compiled during this research.

This survey has shown that the mining engineer requires mainly two rock-material properties, namely, the *uniaxial compressive strength* and the *triaxial strength* of rock. To obtain this information, the design engineer is prepared to supply between 2 to 3 rock specimens for the tests (see Table II). He is interested in the accuracy of results to within 10 to 20 per cent, has a limited budget at his disposal for rock mechanics tests, and is in a hurry. He does not want to worry about all the other factors appearing in Table I. He emphasizes that this approach is dictated by his practical experience, and, in any case, this is only for preliminary design since he designs as he goes along with mining.

These findings confirm that practical experience is still recognized today as the most reliable basis for the design of the majority of rock structures. This agrees with a recent opinion³ that the function of the rock mechanics engineer is not to compute accurately but to judge soundly. In this respect, good practical experience and the correct feel of the problem are essential.

The view should therefore be supported that, while theoretical methods are necessary to provide further understanding of the subject, a practical assistance meaning-

TABLE III
COMPARATIVE ACCURACY OF UNIAXIAL COMPRESSIVE AND POINT-LOAD TESTING

Rock Material	Uniaxial compressive strength				Point-load strength index				
	No. of specimens	Mean MPa	Standard deviation		Core size	No. of specimens	Mean MPa	Standard deviation	
			MPa	%				MPa	%
Sandstone	20	64,5	4,85	7,5	NX BX EX	20 40 30	2,83 3,09 3,36	0,25 0,32 0,28	8,8 10,4 8,3
Quartzite	20	188,4	9,66	5,1	NX BX EX	20 20 20	8,10 9,70 11,96	1,00 1,29 1,38	12,3 13,3 11,5
Norite (weak)	20	253,0	2,25	0,9	NX	20	11,00	1,22	11,1
Norite (strong)	20	313,4	5,21	1,7	NX BX EX	20 20 20	13,13 13,69 15,78	0,89 1,54 0,78	6,8 11,2 4,9

ful to the design engineer should be encouraged, even if it involves the sacrifice of accuracy for simplicity and speed of application.

It is believed that, in the determination of rock properties, such practical assistance should involve the following approach:

- (i) the recommendation of a quick and cheap means for determining the uniaxial compressive strength of rock materials; and
- (ii) the provision of a simple method for estimating the triaxial strength of rock materials.

The above two requirements will now be considered in turn.

DETERMINATION OF THE UNIAXIAL COMPRESSIVE STRENGTH

The uniaxial compressive strength of rock materials is relatively easy to determine, and standardized test procedures are available⁴. This, however, involves laboratory tests that necessitate careful specimen preparation and rather expensive testing apparatus. Attention should therefore be drawn to the study by Broch

and Franklin⁵ of the use of index tests. Their detailed research showed that the point-load strength index, determined on *unprepared* rock cores *in the field* using portable equipment, can provide the uniaxial compressive strength of rock materials with an accuracy well comparable with that derived from laboratory tests.

In view of the promising results obtained by previous researchers, it was decided to investigate this matter further.

The apparatus used for the present study is shown in Plate I. It will

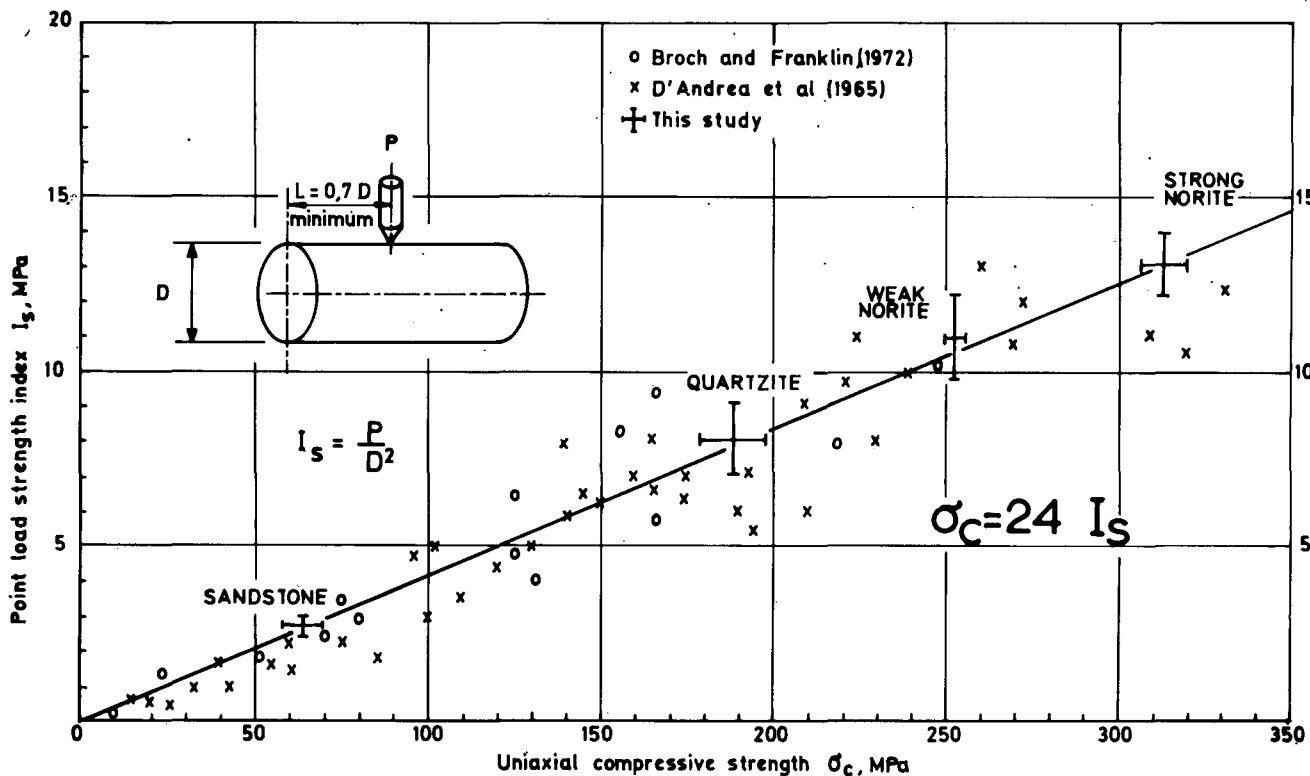


Fig. 1—Relationship between index I_s and strength σ_c for NX core (54 mm dia.)

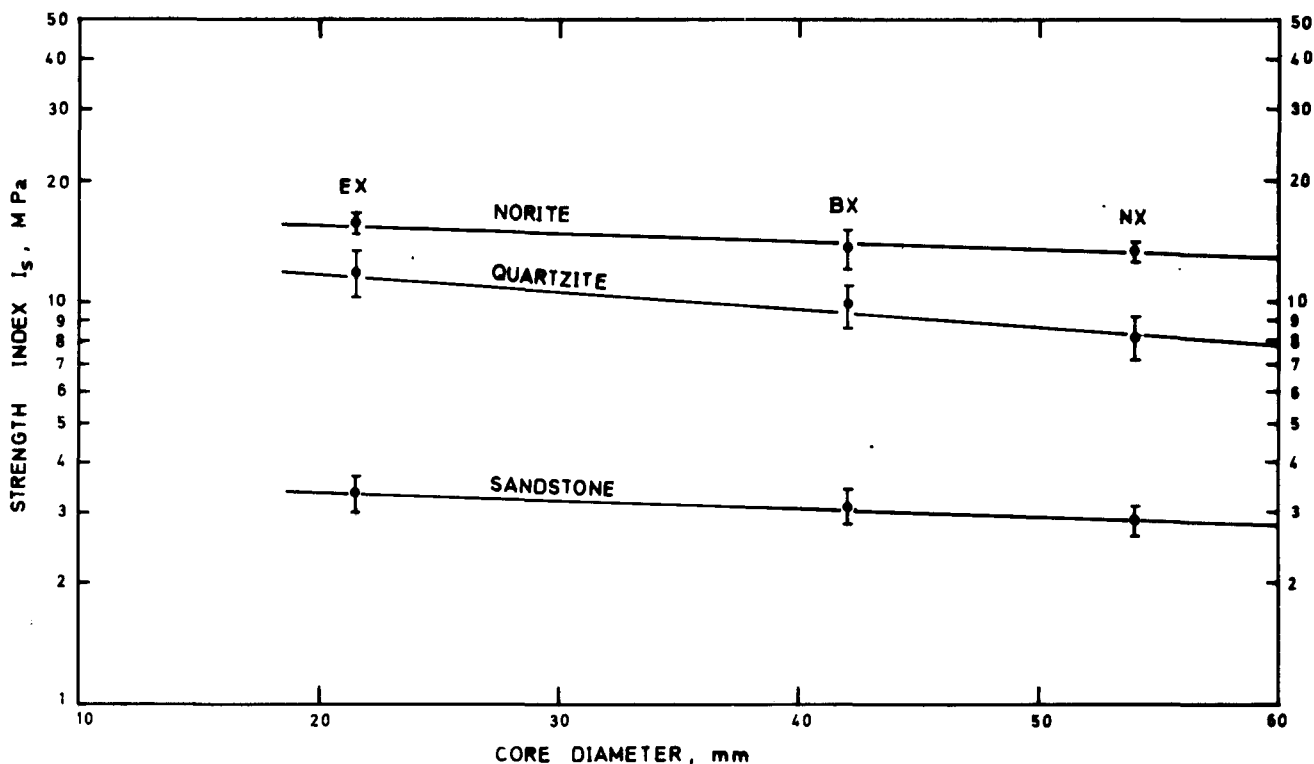


Fig. 2—Size effect in point-load testing

be seen that a piece of core obtained from drilling is compressed between two points. The core fails as a result of a tensile fracture across its diameter, and this tensile nature of failure is an advantage since axial cleavage is the principal mode of rock failure in compression⁶. The point-load strength index is calculated as the ratio of the applied load to the square of the core diameter. The results are listed in Table III and plotted in Fig. 1, from which it will be clear that there is a close correlation between the uniaxial compressive strength and the point-load strength index.

The relationship is as follows:

$$\sigma_c = 24 I_s \dots \dots \dots (1)$$

where σ_c is the uniaxial compressive strength and I_s is the strength index obtained on NX core (54 mm in diameter).

It should be noted that the studied point-load strength index refers to NX core, and thus the results given in Fig. 1 are applicable only to a core diameter of 54 mm. This is important since, if core of

different diameter is used, such as BX (42 mm) or EX (21.5 mm), a correction for the size effect is necessary⁵. This aspect was also investigated during the present study, and Fig. 2 shows how the strength index decreases with increasing core diameter.

The point-load strength index test for the determination of the uniaxial compressive strength of rock materials has been developed to such a degree of reliability that standardized test procedures have been prepared⁷, and the loading apparatus is commercially available. However, the equipment is simple to make, as can be seen from Plate I.

Broch and Franklin proposed⁵ the point-load test as a replacement for the uniaxial compression test and gave the following comparison of the two tests.

Advantages of the point-load test

1. Smaller forces are needed, and a small and portable testing machine can be used.
2. Specimens in the form of core

are used and require no machining.

3. More tests can be made for the same cost.
4. Fragile or broken materials can be tested.
5. Results show less scatter* than those for the uniaxial compression test.
6. Measurement of strength anisotropy is simplified.

Advantages of the uniaxial compression test

7. The testing procedure is better known and evaluated.
8. Results are available for a wide variety of rock types, together with experience on the linking of these results to field performance.

The point-load test has already been used extensively in the U.S.A., U.K., U.S.S.R., and several other European countries. The test is therefore recommended as a simple and convenient method for determin-

*It can be observed from Table III that the present study contradicts this statement.

TABLE IV
CLASSIFICATION OF ROCK MATERIALS FOR STRENGTH

Description	Uniaxial compressive strength, MPa	Point-load strength index, MPa
Very high strength	> 200	> 8
High strength	100-200	4-8
Medium strength	50-100	2-4
Low strength	25-50	1-2
Very low strength	< 25	< 1

ing the uniaxial compressive strength of rock for practical engineering purposes. Table IV lists the corresponding strength ranges for the strength classification of rock materials⁸.

ESTIMATION OF THE TRIAXIAL STRENGTH

While determination of the uniaxial compressive strength of rock is a simple matter, this is not true for the collection of the triaxial strength data, which involves laboratory tests requiring time-consuming specimen preparation and sophisticated testing equipment. Yet, as Table II demonstrates, the triaxial strength data are in demand and there are no simple index tests for this purpose.

It is believed, however, that a case exists for the *estimation* of the triaxial strength of rock from failure criteria.

A criterion of failure is an algebraic expression of the mechanical condition under which a material fails by fracturing or deforming beyond some specified limit. This specification can be in terms of load, deformation, stress, strain, or other parameters.

The search for failure criteria for rock has been conducted for a considerable number of years and, although much progress has been made⁹⁻¹¹, the practical design engineer is still left without a failure criterion that can meet his needs. This is best illustrated by the fact that two-hundred years after Coulomb presented his well-known criterion (1773), it was stated¹²: 'In short, a large amount of research has yet to be done on rock failure criteria, possibly even more than has been published to date'. While this statement is obviously rather pessimistic judging from the amount of material already published on the subject, Jaeger and Cook¹³ justly believe that failure criteria based on

the actual mechanism of fracture, which are more sophisticated than the theories of Coulomb, Mohr, and Griffith, have yet to be developed, since some empirical relations fit the experimental results better.

The author believes that, to meet the immediate needs of the practical rock engineer at the present stage of development of rock mechanics, attention should be directed to empirical criteria for estimating the triaxial strength of rock. Such criteria can be selected by fitting a

suitable equation into experimental data, and they need not have a theoretical basis. They serve to meet the practical requirements of adequate prediction, simplicity of use, and speed of application.

EMPIRICAL CRITERIA FOR TRIAXIAL STRENGTH

The requirement for an empirical triaxial strength criterion is that, from the knowledge of the uniaxial compressive strength of a given rock material, values of the major principal stress, σ_1 , should be predicted, using $\sigma_2 = \sigma_3$ as an input value (the influence of the intermediate principal stress σ_2 can be neglected for practical purposes). The aim is to predict a Mohr strength envelope for a rock material from which one can also obtain its cohesion and friction data.

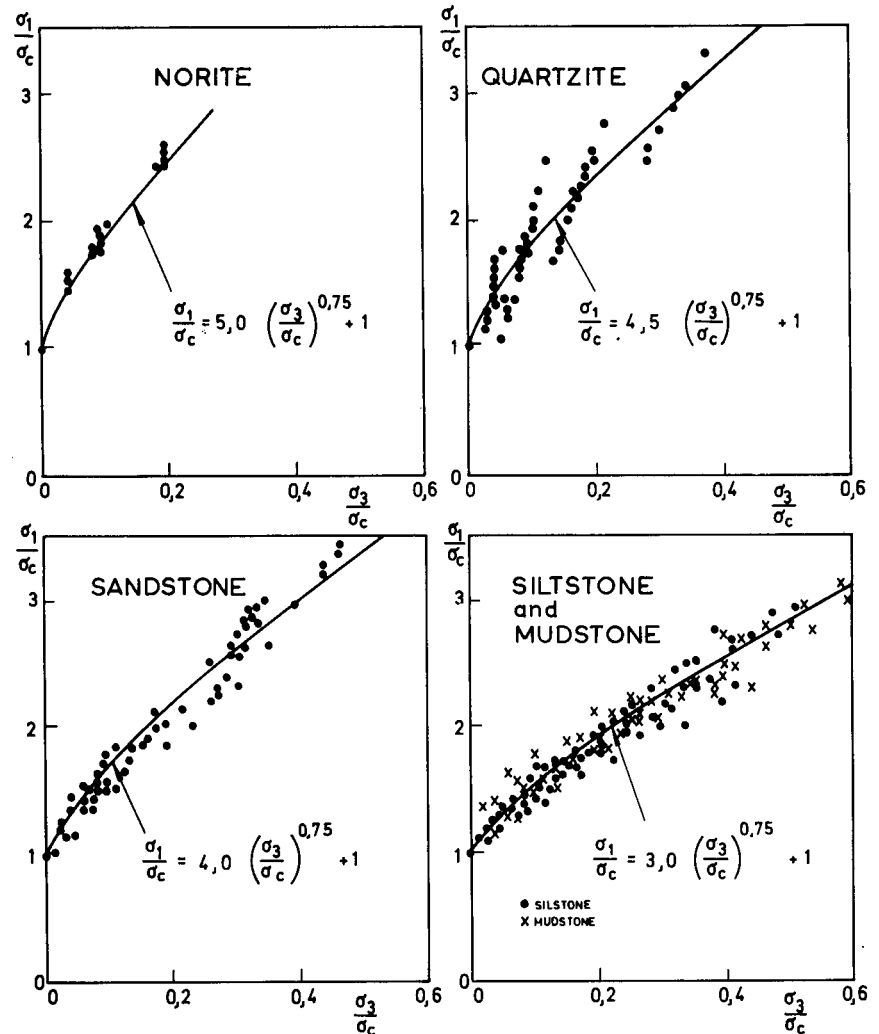


Fig. 3—Stresses at failure in triaxial compression for five rock materials

The present study of the literature published on the subject^{11, 13-18} revealed that the empirical criteria proposed by Murrell¹⁴ and by Hoek¹⁶ are particularly suitable for estimating the triaxial strength of rock. These criteria are not contradictory, and the choice of one instead of the other depends solely on practical convenience.

In 1965, Murrell¹⁴ proposed the following equation:

$$\sigma_1 = F\sigma_3^A + \sigma_c \quad \dots \quad (2)$$

where σ_1 is the major principal stress, σ_3 is the minor principal stress, σ_c is the uniaxial compressive strength, and A and F are constants.

In normalized form, equation (2) can be rewritten as follows:

$$\frac{\sigma_1}{\sigma_c} = k \left[\frac{\sigma_3}{\sigma_c} \right]^A + 1, \quad \text{CRITERION I} \quad \dots \quad (3)$$

where k is a constant.

Normalized form means the expression of the stresses as dimensionless ratios of the uniaxial compressive strength of the material. This has the advantage that, since effects such as the specimen size, environmental conditions, and testing techniques are presumably similar in both numerator and denominator, they are eliminated upon normalization. Furthermore, the direct comparison of a number of tests on the same plot is possible.

In Fig. 3, the triaxial strength results are plotted as σ_1/σ_c versus σ_3/σ_c for five rock types, the two parameters of equation (3) are determined from the experimental data, and Criterion I is fitted for each rock type.

A particularly useful form for an empirical criterion was proposed by Hoek¹⁶ as follows:

$$\frac{\tau_m - \tau_0}{\sigma_c} = B \left[\frac{\sigma_m}{\sigma_c} \right]^C, \quad \dots \quad (4)$$

where τ_m is the maximum shear stress and σ_m the mean normal stress given as:

$$\tau_m = \frac{\sigma_1 - \sigma_3}{2} \quad \sigma_m = \frac{\sigma_1 + \sigma_3}{2} \quad \dots \quad (5)$$

and B , C , and τ_0 are constants. For practical purposes¹⁶, $\tau_0 = \sigma_t$ (uniaxial tensile strength).

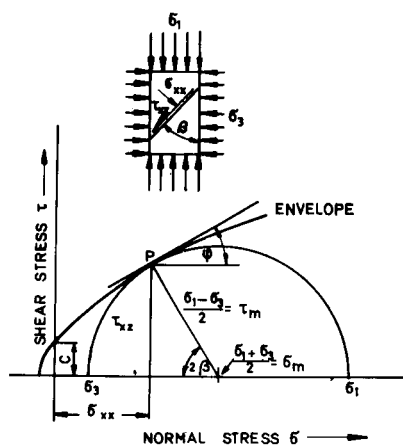


Fig. 4—Mohr's graphical representation of stress

- σ_1 = major principal stress
- σ_2 = minor principal stress
- σ_{xx} = normal stress at failure
- τ_{xz} = shear stress at failure
- C = cohesion
- ϕ = angle of friction
- β = angle of shear failure

The τ_m and σ_m quantities are given physical interpretation (see Fig. 4) as the maximum shear stress acting in the specimen at failure and the normal stress acting on the plane of the maximum shear stress. They are the radius of a Mohr stress circle at failure and the distance from the origin to its centre. A failure criterion expressed in this

way defines the locus of points at the top of Mohr failure circles and is referred to as the maximum shear stress locus.

The suitability of equation (4) for rock materials is best evident from Fig. 5, where the results of triaxial tests on three rock types are plotted on logarithmic scales. For the sake of clarity, other rock types are not shown in Fig. 5. It should be noted that this figure permits a direct evaluation of the constants B and C because B is given by the value of $(\tau_m - \tau_0)/\sigma_c$ when $\sigma_m/\sigma_c = 1$, and the value of C is given by the slope of the straight line through the experimental points. For the complete evaluation of these constants, the value of τ_0 must also be known and, as this is equal to σ_t , the ratio of σ_t/σ_c must be specified in equation (4). However, for practical purposes in rock mechanics, the uniaxial tensile strength, σ_t , is usually one-tenth* of the uniaxial compressive

*This aspect was investigated during the present study, and it was found that the accuracy of prediction of empirical criteria is lowered by only about 1 per cent when compared with the actual values of σ_t and σ_c .

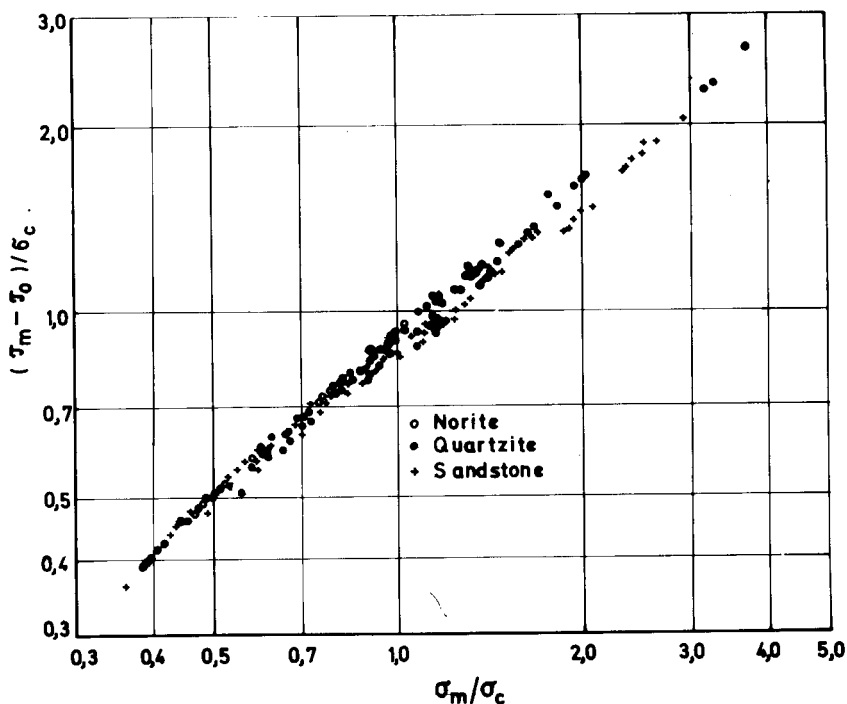


Fig. 5—Experimental data for maximum shear and mean normal stresses at failure for three rock types

strength, σ_c , and equation (4) can be rewritten as follows:

$$\frac{\tau_m}{\sigma_c} = B \left[\frac{\sigma_m}{\sigma_c} \right]^C + 0,1$$

CRITERION II . . (5)

One reason for the choice of the maximum shear stress τ_m and the mean normal stress σ_m , in place of shear and normal stresses τ_{xz} and σ_{xx} , (see Fig. 4) is that the construction of τ_m and σ_m circles and the fitting of a Mohr envelope is, as will be seen later, a simple and reliable graphical operation. This is not the case when fitting an envelope by eye to a set of experimentally determined Mohr circles. Such an envelope often does not include the scatter of experimental values because it is usually fitted to the circles of maximum diameter.

In Fig. 6 the triaxial results are plotted as τ_m/σ_c versus σ_m/σ_c for five rock types, and the two parameters of equation (5) are determined from the experimental data and

Criterion II is fitted for each rock type.

The parameters for Criteria I and II—equations (3) and (5)—were determined from 412 test specimens involving 5 quartzites (91 specimens), 5 sandstones (109 specimens), 1 norite (35 specimens), 4 mudstones (86 specimens), and 4 siltstones (91 specimens). The results are given in Table V, which also includes the average prediction errors for each rock type. These prediction errors were calculated as the difference between observed σ_1 and predicted σ_1 expressed as positive percentage of the predicted value.

It is clear from Table V that both criteria yield prediction errors that are very acceptable for practical engineering purposes. Thus, the actual application of these criteria depends solely on practical convenience.

It is recommended that Criteria I and II should be used as follows:

—Criterion I is employed when the individual values of σ_1 are

required for a given σ_3 , as will be the case when the virgin stress field in a rock mass is known and its comparison with the material strength under triaxial conditions is needed. For this purpose, Criterion II is not suitable.

—Criterion II is employed whenever a full Mohr envelope is required and values of cohesion and friction of a rock material are needed. For this purpose, Criterion I is not recommended because it is valid only for compression.

An example of how Criteria I and II can be applied in practice will now be presented.

PRACTICAL EXAMPLE

Consider a problem of estimating the uniaxial and triaxial strengths of a quartzite, for which point-load index tests on NX cores (diameter $D=54$ mm) gave an average failure load $P=23,1$ kPa. In the design of an underground mining excavation

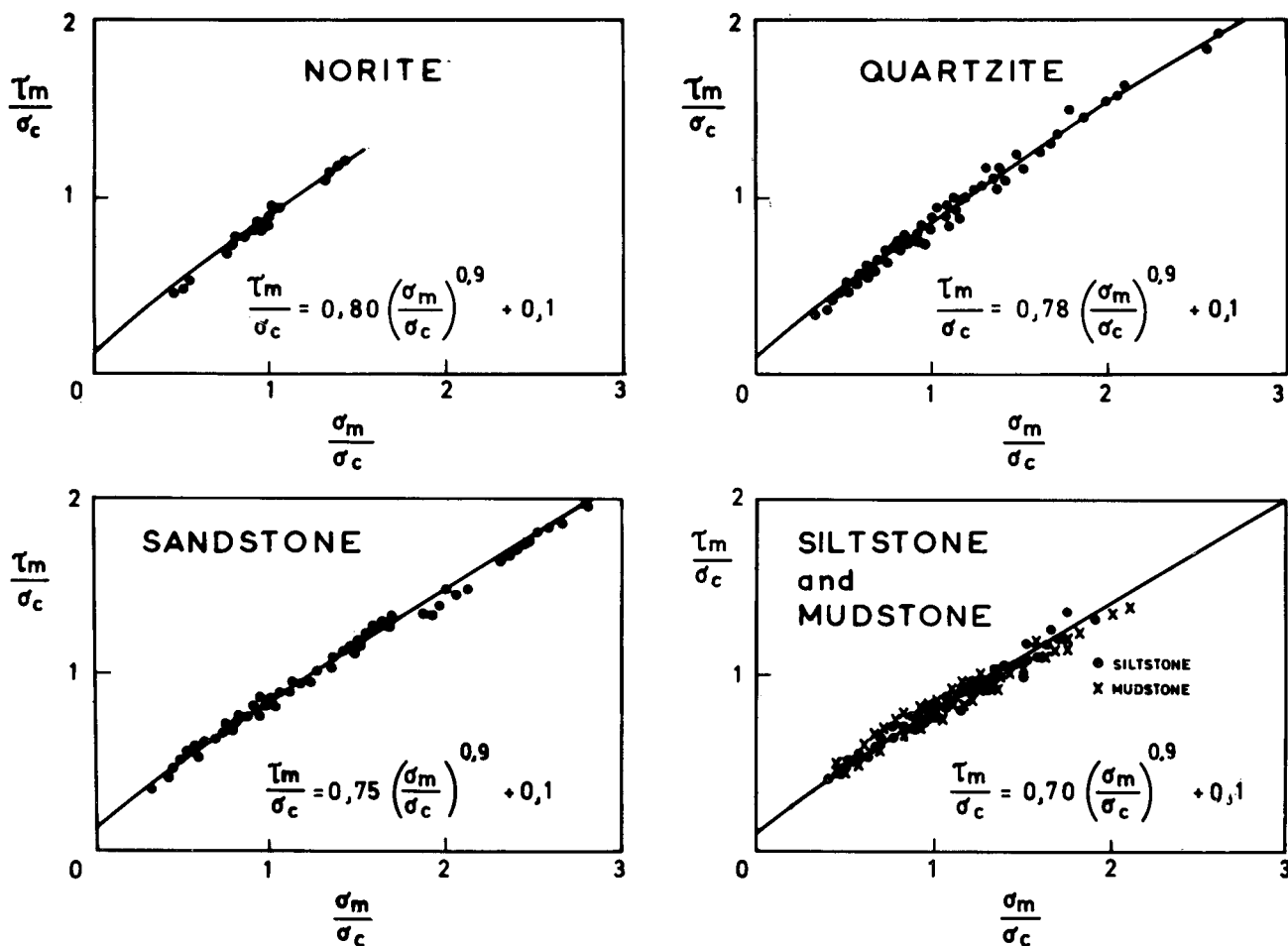


Fig. 6—Relations between the maximum shear and mean normal stresses at failure for five rock materials

TABLE V
EMPIRICAL CRITERIA FOR TRIAXIAL STRENGTH OF ROCK MATERIALS

CRITERION	I		II	
	$\frac{\sigma_1}{\sigma_c} = A \left[\frac{\sigma_3}{\sigma_c} \right]^{0,75} + 1$		$\frac{\tau_m}{\sigma_c} = B \left[\frac{\sigma_m}{\sigma_c} \right]^{0,9} + 0,1$	
ROCK TYPE				
Norite	A = 5,0	Error: 3,6%	B = 0,8	Error: 1,8%
Quartzite	A = 4,5	Error: 9,2%	B = 0,78	Error: 3,2%
Sandstone	A = 4,0	Error: 5,8%	B = 0,75	Error: 2,3%
Siltstone	A = 3,0	Error: 5,6%	B = 0,7	Error: 4,2%
Mudstone	A = 3,0	Error: 6,1%	B = 0,7	Error: 6,6%
ALL ROCK TYPES	A = 3,5	Error: 10,4%	B = 0,75	Error: 8,3%

involving this rock material at a depth of 3000 m below surface and subjected to the virgin stresses of 70 MPa in the vertical direction and 50 MPa in each horizontal direction, the following estimates are required:

- uniaxial compressive strength,
- triaxial strength for the *in situ* confining pressure $\sigma_3=50$ MPa, and
- Mohr envelope.

Solution

- The point-load strength index $I_s = P/D^2 = 23\ 100 / (54 \times 54) = 7,92$ MPa. The uniaxial compressive strength $\sigma_c = 24 I_s = 24 \times 7,92 = 190$ MPa.
- Criterion I for quartzite is as follows:

$$\frac{\sigma_1}{\sigma_c} = 4,5 \left[\frac{\sigma_3}{\sigma_c} \right]^{0,75} + 1$$

$$= 4,5 \left[\frac{50}{190} \right]^{0,75} + 1 = 2,65, \text{ and}$$

$$\sigma_1 = 2,65 \times \sigma_c = 2,65 \times 190 = 503,5 \text{ MPa.}$$

The triaxial strength of this quartzite is far higher than the acting vertical *in situ* stress of 70 MPa.

- Criterion II for quartzite is as follows:

$$\frac{\tau_m}{\sigma_c} = 0,78 \left[\frac{\sigma_m}{\sigma_c} \right]^{0,9} + 0,1.$$

The above relationship is plotted in Fig. 7 and is marked Criterion II. Since this criterion defines the

locus of points at the top of Mohr failure circles, such circles are drawn in Fig. 7 and a Mohr envelope is next fitted.

Fig. 7 provides the material cohesion of quartzite as $C_o = 0,2 \sigma_c = 0,2 \times 190 = 38$ MPa, and the angle of friction for quartzite as $\phi_o = 45^\circ$ at $\sigma_m/\sigma_c = 70 + 50 / (2 \times 190) = 0,32$ (calculated from the virgin stresses).

CONCLUSION

A practical approach has been presented for the prediction of the strength behaviour of rock materials. It has been shown that an estimate

of the triaxial strength of rock materials can be made by means of two empirical strength criteria that have an accuracy sufficient for practical purposes. The only input required for these criteria is the uniaxial compressive strength. This is simply determined in the field from the described point-load strength index obtained on unprepared rock cores.

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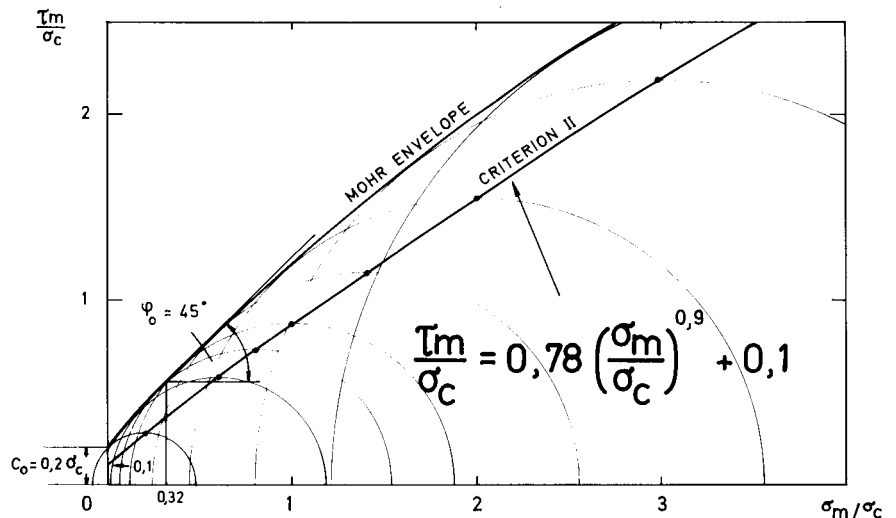


Fig. 7—Mohr stress diagram for quartzite

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Company affiliates

The following members have been admitted to the Institute as Company Affiliates.

AE & CI Limited.

Afrox/Dowson and Dobson Limited.
Amalgamated Collieries of S.A. Limited.

Apex Mines Limited.

Associated Manganese Mines of S.A. Limited.

Blackwood Hodge (S.A.) Limited.

Blyvooruitzicht G.M. Co. Ltd.

Boart & Hard Metal Products S.A. Limited.

Bracken Mines Limited.

Buffelsfontein G.M. Co. Limited.

Cape Asbestos South Africa (Pty) Ltd.
Compair S.A. (Pty) Limited.

Consolidated Murchison (Tvl) Goldfields & Development Co. Limited.

Doornfontein G.M. Co. Limited.

Durban Roodepoort Deep Limited.

East Driefontein G.M. Co. Limited.

East Rand Prop. Mines Limited.

Free State Saaiplaas G.M. Co. Limited.

Fraser & Chalmers S.A. (Pty) Limited.

Gardner-Denver Co. Africa (Pty) Ltd.

Goldfields of S.A. Limited.

The Grootvlei (Pty) Mines Limited.

Harmony Gold Mining Co. Limited.

Hartebeesfontein G.M. Co. Limited.

Hewitt-Robins-Denver (Pty) Limited.

Highveld Steel and Vanadium Corporation Limited.

Hudemco (Pty) Limited.

Impala Platinum Limited.

Ingersoll Rand Co. S.A. (Pty) Ltd.

James Sydney & Company (Pty) Limited.

Kinross Mines Limited.

Kloof Gold Mining Co. Limited.

Lennings Holdings Limited.

Leslie G.M. Limited.

Libanon G.M. Co. Limited.

Lonrho S.A. Limited.

Lorraine Gold Mines Limited.

Marievale Consolidated Mines Limited.

Matte Smelters (Pty) Limited.

Northern Lime Co. Limited.

O'okiep Copper Company Limited.

Palabora Mining Co. Limited.

Placer Development S.A. (Pty) Ltd.

President Steyn G.M. Co. Limited.

Pretoria Portland Cement Co. Limited.

Prieska Copper Mines (Pty) Limited.

Rand Mines Limited.

Rooiberg Minerals Development Co. Limited.

Rustenburg Platinum Mines Limited (Union Section).

Rustenburg Platinum Mines Limited (Rustenburg Section).

St. Helena Gold Mines Limited.

Shaft Sinkers (Pty) Limited.

S.A. Land Exploration Co. Limited.

Stilfontein G.M. Co. Limited.

The Griqualand Exploration and Finance Co. Limited.

The Messina (Transvaal) Development Co. Limited.

The Steel Engineering Co. Ltd.

Trans-Natal Coal Corporation Limited.

Tvl Cons. Land & Exploration Co.

Tsumeb Corp. Limited.

Union Corporation Limited.

Vaal Reefs Exploration & Mining Co. Limited.

Venterspost G.M. Co. Limited.

Vergenoeg Mining Co. (Pty) Limited.

Vlakfontein G.M. Co. Limited.

Welkom Gold Mining Co. Limited.

West Driefontein G.M. Co. Limited.

Western Deep Levels Limited.

Western Holdings Limited.

Winkelhaak Mines Limited.