



Variance relationships between the masses, grades and particle sizes for gold ores from the Witwatersrand

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Synopsis

A model relating the variance of gold assays in different size fractions of products from crushing a low grade ore from the Witwatersrand to the mass of the samples was established using multiple linear regression. This model was compared to models for other ores from the Witwatersrand determined from sampling tests that followed the procedures advocated by François-Bongarçon, which follow the well-known principles from Gy.

All the models had a slope of 1 for the relationship between the log of the mass variance product to the log of the particle size.

The model was extended to include a parameter for grade by using models on ores with grades varying from 0.5 g/t to 60 g/t gold.

The relationships between variance, mass, grade and particle size of samples of gold from the Witwatersrand are determined by the macroscopic distribution of the ore, which is present in a quartz conglomerate, in a barren matrix. The microscopic features of the distribution of gold particles, which are estimated from various factors given in the original Gy equation, would only play a minor role. Therefore the relationships need to be modelled using experimental data.

The overall model is proposed as a working model to determine sampling strategies in terms of the mass of samples, the particle sizes to which the samples should be crushed before splitting and the mass of sub-samples that are required.

An example of applying the model to sampling and sample preparation for a large gold mine is given and the variances that could be anticipated are calculated.

The model is intended only as a starting point. The data collected from operating the sampling plant would be analysed statistically using components of variance analysis to measure the variances that are achieved in practice. These variances would be used to refine the models and adjust the crushing and splitting parameters and optimize the numbers of samples sent to the labs for analysis.

Introduction

In many situations there is a need to estimate the precision that can be expected for measurements of the grade of gold ores. Typically, samples of ore of a relatively large mass are taken. These samples are crushed to reduce the particle size and sub-samples of the crushed product taken. These in turn, may be further reduced in particle size and sub-sampled again in successive stages until there

is a sample that is submitted to the laboratory for analysis. There are errors at each of the crushing and splitting stages and also in the final assay in the lab.

It is necessary to have estimates of the precision of all these steps. Precision is the repeatability of the results and is measured in terms of standard deviation and variance, as defined statistically and not as a difference as used in accounting procedures. In sampling a key issue is the variance for samples with different particle sizes and grade. For example, it is considered that samples with large particle sizes have a high variance and it is necessary to take large samples to obtain what are loosely called 'Representative Samples'. The variance is a quantitative measure of how representative a sample is.

The accuracy of the final result is a different question to that of variance or precision and is not considered in this paper. Accuracy is the closeness of the final result to the true value, which is not known. Accuracy is assured by correct design of samplers and by the absence of bias in splitting and in laboratory measurements.

The usual method for estimating the variances of samples with different particle sizes involves a theoretical consideration of the fundamentals of sampling, using the so-called Gy formula after appropriate calibration by sampling experiments to establish the parameters for particular ores. These methods have been used to determine the variance size relationships for Witwatersrand gold ores.

In this paper, the variances with different particle sizes from a crushed low grade ore derived from waste rock dumps are

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determined. The results from these measurements are compared to published results using the usual method with the main objective of defining strategies for preparing run-of-mine samples in terms of the crushing and sub-sampling procedures.

Usual method for calculating variance size relationships

A well-known theoretical sampling model is that of Gy¹. A description of the model is given by Assibey-Bonsu². The model is based upon the equiprobable sampling model and assumes the random distribution of mineral particles in the host matrix of specific particle size. The general form of Gy's model, for lots where the sample mass is much smaller than the lot mass, is:

$$\sigma_R^2 = f \cdot g \cdot c \cdot l \cdot d_n^3 / M \quad [1]$$

Where :

- σ_R^2 = Relative variance of sample
- d_n = Nominal size of the fragments in the sample. This is the maximum particle size in the lot to be sampled. In practice, d_n is taken as the mesh size that retains 5% of the lot being sampled and is measured in centimetres
- M = Mass of the sample
- f = particle shape factor which is an index varying in most cases between 0 and 1. For most ores a value of 0.5 is used
- g = Granulometric factor (or grain size distribution factor) and takes the values between 0 and 1. For most ores g is taken as 0.25
- c = mineralogical composition factor and is the volume proportion of the gold in the host matrix. It is calculated from the grade of ore in the sample and the density of gold
- l = liberation factor. In the original Gy formula the liberation factor was the ratio of the liberation size of the mineral and the nominal size of the fragments in the sample. However, Gy made an empirical adjustment whereby the liberation factor was taken as the square root of this ratio. $l = (d_o/d_n)^{0.5}$
- d_o = The liberation size for the mineral particles i.e. the maximum particle diameter that ensures complete liberation of the mineral. It equates to the particle size of the minerals.

Gy gives estimates for the various factors. For typical gold ores, using the suggested values for f and g and the liberation factor. $l = (d_o/d_n)^{0.5}$ the formula gives large masses of samples that would be required. Assibey-Bonsu calculated that 2507 tons of sample would be required of a gold ore with a grade of 5 g/t. Bartlett and Hawkins³, for a 6 g/t ore and a relative precision of 8%, calculated that a 900 ton sample is needed.

In the case of gold ores from the Witwatersrand the mineralogy is complex (Hallbauer⁴) and this makes the estimation of the factors for the Gy formula to be problematic.

For this reason, the models need to be calibrated by sampling experiments to determine liberation and other factors (Bongarçon⁵). The methods advocated by Bongarçon

have become the usual method for determining the size grade relationships.

Calibration of formula

The Gy formula, Equation [1], can be generalized by taking logs and substituting the exponent 3, or 2.5 in a modification to the original formula, for the nominal particle size in the sample for a more general exponent, α . The equation then becomes:

$$\text{Log}(M * \sigma_R^2) = \text{Log } K + \alpha \text{Log}(d_n) \quad [2]$$

The constant K includes all the factors which affect the relationships. K may include the factors, l , c , g and f in the original Gy equation but could also incorporate other and possibly different terms which are not known. From a practical viewpoint, it is not important to know these individual factors.

François-Bongarçon⁶ explored the relationships between variance, mass of sample and particle size of the samples using a series of samples, which were crushed to specific sizes before being split, their masses measured and the variances determined from assays. By plotting $\text{Log}(M * \sigma_R^2)$ against $\text{Log}(d_n)$ his work showed that the slope of the line, α , was dependent upon the particular ore that was used but was closer to 1.5 than 2.5 or 3 as predicted by the Gy formulae.

Using a value of 1.5 for α , Assibey-Bonsu found that a sample mass of 1.4 tons would be required compared to 2507 ton sample predicted with the Gy formula with α set to 2.5. A more realistic figure is 1.4 tons and is about the mass of a primary sample for run-of-mine sampling with particle sizes of 250 mm.

From the values of K and α , the variance of a sample of a specific mass and particle size can be determined for the particular ore for which the test was performed.

Statistical model for the variance size of sample relationships

An alternative to the calibration tests where special experiments, in which the ore is crushed to specific sizes and the variances determined, is to construct mathematical models directly from assay and mass data for ranges of screen size fractions. This approach uses a general form of equation where variance is taken to vary with particle size, mass and grade of the sample.

Experimental data for variance, grade and size of sample

A set of data on the particle size range and gold assays for a pilot scale trial on the crushing of a low grade gold ore was used to generate a statistical model relating the variance of the assays at particular sizes to the mass of the samples. The ore was the accumulated waste over many years from operation of a large gold mine in the West Witwatersrand. The gold would be present as fragments of ore from the range of reef types.

Experimental methods

The objective for the trials on the crusher which was being

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tested was to determine the proportion of the gold in the ore that reported to the various undersize fractions. For the test programme there were eight different settings of the crusher and the complete range of particle sizes were collected for each of the settings. Three passes were generally made through the crusher at each setting. This allowed for the collection of a considerable amount of data for assessing the grade variance relationships.

Approximately 250 kg samples of the products from crushing at the eight different settings were produced by coning and quartering. These were screened at about 30 mm, 20 mm, 14 mm, 10 mm and 6 mm to give the oversize fractions, which were weighed. The minus 6 mm material was retained. A rotary splitter used to split out three one-quarter portions from each of the oversize fractions. The sub-samples of the coarse fractions were prepared by crushing to 1.8 mm and split on a rotary splitter before being sent to the lab for pulverizing and fire assay. The variances for the gold assays at the different screen sizes were determined from the three sub-samples.

A rotary splitter was used to split out four portions of about 500 gms from the minus 6 mm material. Three of the four portions were screened from 6 mm down to minus 75 microns. The masses of the various size fractions were measured and then sent to the lab for triplicate fire assay.

Mean data

The mean values, at the various mesh sizes for the mass of samples, for the mean grades, the relative variance and the coefficient of variation are given in Table I.

CV% is the coefficient of variation expressed as a percentage. It is equivalent to the relative standard deviation (RSD).

Table I shows:

- The CV% is high and increases with the particle size but decreases with sample mass
- The gold grade is higher in the finer size fractions.

Modelling of errors

A general form of the variance equation, which includes both particle size and gold grade is:

$$\sigma_R^2 = R.d_n^\alpha / M.a^\beta$$

This form is similar to the form used for the Gy Equation [1].

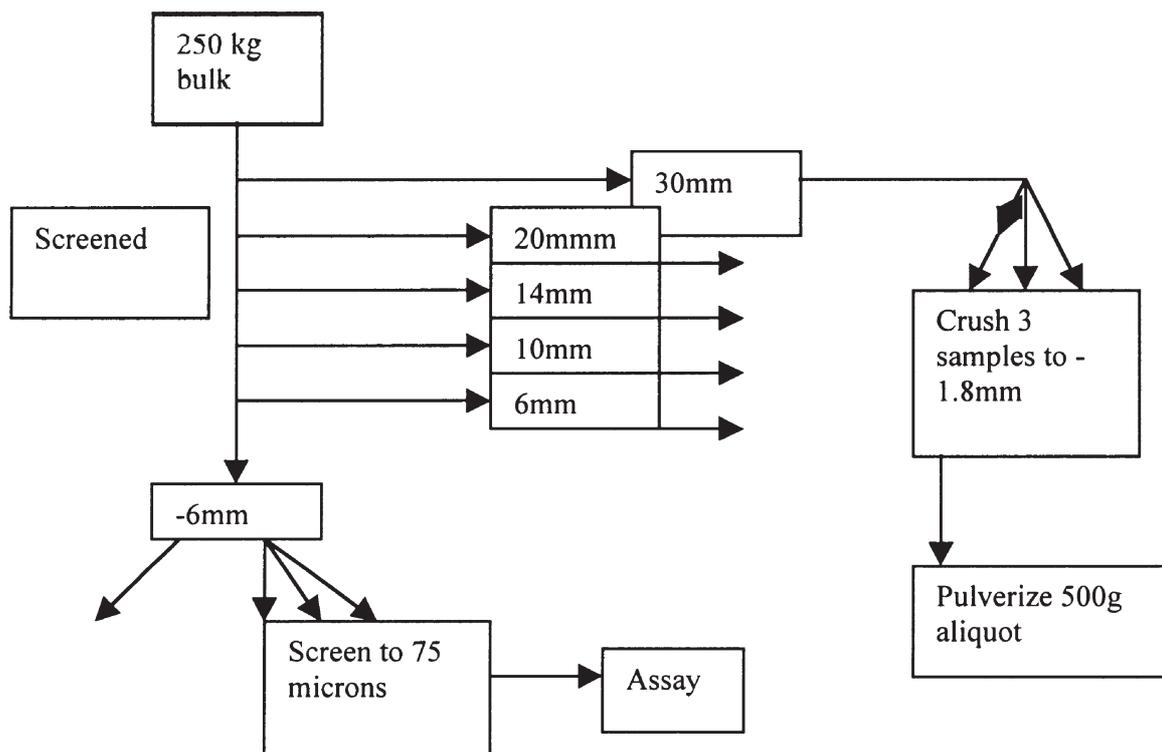
Where:

$$\sigma_{R^2} = \text{Relative variance of sample}$$

Table I

Mean statistics for size fractions

Size (cms)	Mass (gms)	Gold g/t	Relative variance	CV%
3.14	8703.75	0.1725	0.02139	14.6
2	16966.25	0.246	0.12795	35.8
1.4	7943.75	0.2715	0.06474	25.4
1	4280	0.0425	0.13536	36.8
0.6	2685	0.18383	0.1998	44.7
0.5	455	0.1595	0.35098	59.2
0.475	165.18	0.16667	0.24931	49.9
0.336	176.77	0.35733	0.45335	67.3
0.235	119.43	0.2355	0.31069	55.7
0.118	169.45	0.35742	0.30088	54.9
0.0425	187.25	0.39833	0.20223	45.0
0.0075	183.43	0.82717	0.03257	18.0
0.0038	35.2	1.85267	0.00977	9.9



Sketch of sample splitting and screening procedure

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d_n = Nominal size of the fragments in the sample.
This is the maximum particle size in the lot to be sampled. In practice, d_n is taken as the mesh size that retains 5% of the lot being sampled and is measured in centimetres

M = Mass of the sample
 α = exponent for particle size
 a = assay of gold in g/t
 β = exponent for assays
 R = Residual constant.

This equation is converted to logs.

$$\text{Log}(\sigma_R^2 * M) = \text{Log}R + \alpha \cdot \text{Log}(d_n) + \beta \cdot \text{Log}(a)$$

The raw data on the mesh sizes, grades, masses and variances was analysed by regression analysis using the General Linear Model, GLM procedure in SAS⁷.

The output from the statistical analysis is presented as an appendix.

A graphical representation of the particle size, variance relationship is given as Figure 1.

For a multivariate analysis of the data as presented in the appendix:

- ▶ The dependent variable is the log(mass*variance) and the independent variables are the log of the size and the log of the mean gold grade in each of the size fractions
- ▶ The model is significant statistically with an F value of 37.05 and a probability that the relationships are purely due to chance of less than 0.0001. The correlation coefficient is 75.5%
- ▶ The intercept is 5.56 and it is significantly different from zero (F value 39.62 probability of less than 0.0001)
- ▶ The slope for log size is 1.05 and is significant
- ▶ The slope for log of mean gold grade is not significant.

On the basis of the statistical model the parameters for the relationship between the log(variance*mass) and log size are:

$$\text{Intercept (Log}K) = 5.56(\pm 0.74)$$

$$\text{Slope } (\alpha) = 1.05(\pm 0.17).$$

Because the coefficient for the effect of grade was not significant, the intercept (log K) includes the effect of grade and this intercept is equivalent to the intercept K in the methodology proposed by François-Bongarçon. (Equation [2]).

Sampling model for different grades of ore and particle sizes

Afewu and Lewis⁸ following the procedures advocated by François-Bongarçon crushed two separate Witwatersrand gold ores, one a low-grade ore of about 5 g/t and the second with a grade of about 60 g/t, to different sizes and determined the relative variances.

These were modelled to give the constant K and the slope, α , for the calibration of the sampling model. In some other work at Randfontein Estates, the variances at 15 mm, 6 mm and 75 microns was used to construct the same type of model.

The grades, slopes and intercepts for these results and for the work on waste crushing are presented in Table II.

The average value of α is 1. This is taken to be the mean value for the ores and waste considered.

The relationship between K , the constant when the grade is not considered, and R , the residual constant when grade is considered, is:

$$K = R / a^\beta$$

Taking logs:

$$\text{Log } K = \text{Log } R + \beta \text{ log } a$$

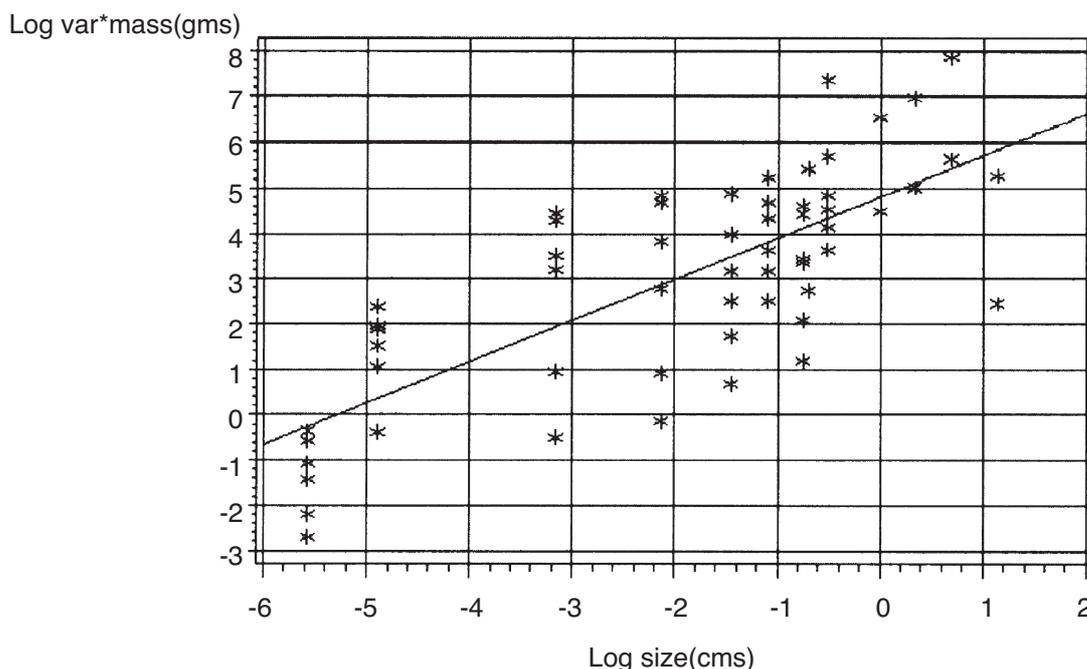


Figure 1—Relationship between relative variance, mass and particle size for waste gold ore

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Table II
Slope and intercept for different Witwatersrand ores

Source	Grade g/t	Constant K	Slope
Screened data	0.5	260	1.04
Randfontein Estates	4.83	62	0.94
Afewu and Lewis	5.19	94	1.01
Afewu and Lewis	62.9	15.7	1.13
Average slope			1.03

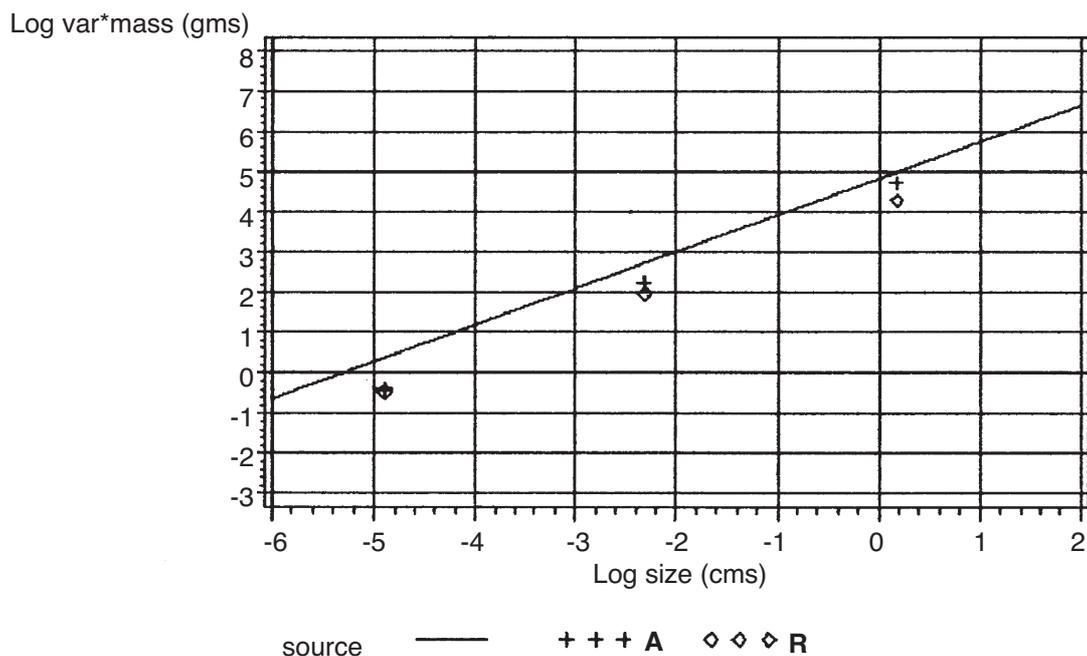


Figure 2—Comparisons of relationships with closely screened data and with cumulative under-size data. A=Afewu and Lewis low grade ore. R=Randfontein Estates regression line for closely screened data

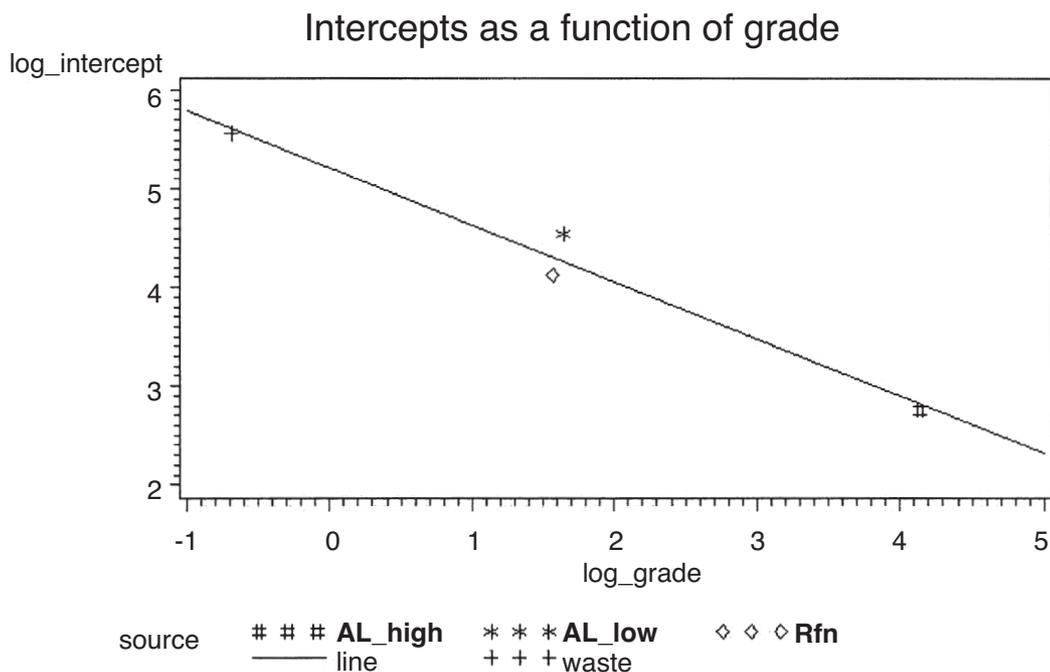


Figure 3—Relationship between constant K and ore grade. AL High = Afewu and Lewis high grade ore. AL low = Afewu and Lewis low grade ore Rfn = Randfontein

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Figure 3 is a plot of $\text{Log } K$ versus log grade, using the data in Table III. The slope of this line gives β , and the intercept $\text{log } R$.

This relationship was modelled using regression analysis with the model:

$$\text{Log } K = \text{Log } R + \beta \text{ log } a$$

From a multivariate analysis of the data in Table III:

- ▶ The dependent variable is the log of the intercept K and the dependent variable is the grade of the ore
- ▶ The model is significant statistically with an F value of 67.12 and a probability that the relationships are purely due to chance of less than 0.0001. The correlation coefficient is 98.4%
- ▶ The log (intercept R) is 5.21 and it is significantly different from zero (T value 30.82 probability of 0.0011 which is significant)
- ▶ The slope for log grade is -0.58 and is significant.

The parameters for this grade model are:

Intercept ($\text{Log } R$)	= 5.21 (± 0.17)
R	= 183
Slope β	= -0.58 (± 0.07).

Working model for sampling Witwatersrand gold ores

On the basis of the statistical work on the size variance relationships for low grade ore and the agreement between this work and that with other ores from the Witwatersrand the following parameters are suggested for a working model to define sampling strategies:

$$R = 183$$

$$\alpha = 1$$

$$\beta = -0.58.$$

Using these parameters:

$$\sigma_R^2 = 183 \cdot d_n^1 / M \cdot a^{-0.58}$$

In log terms:

$$\text{Log}(\sigma_R^2) = 5.21 + \text{Log}(d_n) - \text{Log}(M) - 0.58 \text{Log}(a)$$

Where:

R	=	Constant
α	=	Exponent for particle size
β	=	Exponent for gold grade
σ_R^2	=	Relative variance
d_n	=	Nominal size of particles in cm
M	=	Mass of sample gms
a	=	Gold grade g/t.

The constant R contains various unmeasured, and possibly un-measurable quantities possibly related to the mineralogy of the gold ore, to the distribution of large agglomerates of reef within waste or other factors. However, from a practical point of view, for example to define sampling strategies, these questions are not relevant.

Example of a sampling strategy for gold ores using the Witwatersrand gold model

As an example of the application of the model, the errors in

sample preparation for a bulk run-of-mine sample at a large gold mine is considered. There is an additional variance associated with the analysis in the laboratory and this needs to be considered separately, particularly as chemical analysis has a major impact on accuracy.

This mine obtains Witwatersrand gold ore from a number of different shafts and treats this in several plants. The ore is largely from reefs where the gold is distributed over the stope width and is not in the form of narrow high grade layers. Go-belt samples are taken at 150 ton intervals from the belts delivering ore from the shafts. The mass of each sample increment is about 50 kgs and may range from 30 kgs to 100 kgs. Depending on the tonnage at the shaft, there are between 13 and 26 individual samples per day. The top particle size of the samples is 300 mm, with any particles larger than this being broken by hammer through grizzlies. The daily bulk samples are collected in containers and these transported to a central facility.

At this facility, the bulk samples will be crushed in two stages, with 14x24 and 8x5 crushers, from the 300 mm size to minus 25 mm and a sub-sample split out at this intermediate stage. The sub-sample would be dried and then put through a 5x3 crusher to reduce the particle size further before splitting out a sample to be pulverized before splitting a final sample for the lab.

The relative variances and CV% that would be achieved for different masses of sample, at different particle sizes have been calculated using the statistical model for a 5 g/t ore.

These are presented in Table III.

In Table III the relative variances and the coefficients of variation (CV%) for the different masses and particle sizes are presented

There are six separate sampling stages namely:

Model $\text{Log}(\text{mass}^2 \text{variance}) = \text{log } R + \text{log}(\text{Particle size}) - \text{log}(\text{Sample mass} \cdot 0.58)$ (GR = 183 Grade = 5 g/t Particle size in cms Masses in grams)					
	Sample mass kgs	Size cms	Log rel. variance	Relative variance	CV%
Bulk	600	30	-5.62696	0.003599	6.0
	1000	30	-6.13779	0.00216	4.6
	4000	30	-7.52408	0.00054	2.3
Intermediate	50	4	-5.15696	0.005759	7.6
	100	4	-5.85011	0.00288	5.4
Sample 1	10	2.5	-4.01752	0.017997	13.4
	20	2.5	-4.71067	0.008999	9.5
	30	2.5	-5.11614	0.005999	7.7
	50	2.5	-5.62696	0.003599	6.0
	200	2.5	-7.01326	0.0009	3.0
6mm too coarse	1.5	0.6	-3.54752	0.028796	17.0
	2.5	0.6	-4.05835	0.017278	13.1
	10	0.6	-5.44464	0.004319	6.6
3mm subsample	1.5	0.3	-4.24067	0.014398	12.0
	2.5	0.3	-4.75149	0.008639	9.3
	3	0.3	-4.93381	0.007199	8.5
	10	0.3	-6.13779	0.00216	4.6
Pulverize for assay	0.1	0.0075	-5.2215	0.005399	7.3
	0.1	0.0038	-5.9014	0.002736	5.2
	0.5	0.0075	-6.83093	0.00108	3.3
	0.5	0.0038	-7.51084	0.000547	2.3

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- ▶ **Bulk sample**—This is the sample that is delivered in containers and the mass of sample varies between 600 kgs and 4 tons depending on the throughput from the shafts. The particle size of the sample is minus 30 cm. As can be seen on the Table, the CV% at 600 kgs is 6.0% and at 4 tons it is 2.3%. The mass of the bulk sample is a function of the number of belt cuts that are taken with the go-belt sampler. It is currently set to take one cut every 150 tons. A bigger bulk sample would be obtained with more frequent cuts. However, the CV% of 4.6% for the normal 1 ton bulk sample is adequate.
- ▶ **Intermediate sample**—This is a sub-sample produced by crushing the bulk sample to a particle size of minus 4 cm. It is not proposed to crush only to 4 cm but there could be circumstances where this coarse crush is obtained because the crusher becomes blocked with wet ore if it is set at 2.5 cm. With the 4 cm crush it would be necessary to take a sample of 50 kgs. For a 50 kg intermediate sample from the normal 1 ton bulk sample a 20:1 splitting ratio is required. The 50 kg sample would be dried before submitting it to the next stage of crushing in a 5x3 crusher from 4 cm to 0.3 cm. At 50 kgs and a particle size of 4 cm the CV% is 7.6%.
- ▶ **Sub-sample 1**. In the situation that the ore is sufficiently dry to be crushed in the two stages, using the 14x24 and 8x5 crushers, to a particle size of 2.5 cm, it is necessary to take a sample of 30 kgs instead of 50 kgs needed for the 4 cm particle size. The sampling needs to be organized so as to take a sub-sample of about 30 kg irrespective of the mass of the bulk sample, which is normally 1 ton but can vary from 600 kg to 4 tons. A way of doing this would be to have a rotary sampler with the appropriate radius and fitted with four or more containers on radial arms. During the sampling process all of the containers would collect sample but at the end only one, two or more containers would be emptied to make up the 30 kgs required for the drying and subsequent stages. At 2.5 cm particle size and a mass of 30 kgs the CV% is 7.7%, which is adequate
- ▶ **6 mm sub-sample**—The 50 kg sub-sample at 4 mm or the 30 kg sample at 2.5 cm would be crushed after drying. With this size crusher, the product is normally minus 0.6 cm. However, Table V shows that for a mass of 2.5 kg, the CV% is 13.1% as compared to 9.3% for material crushed to minus 0.3 mm. This CV% of 13.1% is too high and it is recommended that a particle size of 0.3 mm should be used.
- ▶ **3 mm sub-sample**—It is important that setting of the 5x3 crusher is such that 0.3 cm product can be maintained. A 10:1 split is required to produce a 3 kg sample from the 30 kg from sub-sample 1. It is recommended that a rotary sampler with two containers be used. If the 50 kg from the intermediate stage is treated, only one container would be used. The CV% for 3 kg at a particle size of 0.3 cm is 8.5%, which is adequate.
- ▶ **Pulverized for assay**—The 3 kg samples could be pulverized using vertical spindle pulverizers to a particle size of minus 75 microns. The pulverized product would be split on a rotary splitter to produce 500 g samples. In normal routine two of these samples would be submitted to the lab for triplicate assay. The CV% for splitting out 500 g samples at a particle size of 75 microns is 3% This is satisfactory.
- ▶ **Assay variance**—However, in addition to the variance of splitting there is the variance of the assay itself. The assay variance varies from lab to lab. Typical variance of assays is given in Table IV. SARM53 is a standard gold sample and duplicate wastes are assays of low grade gold ore.

Calculation of overall variance

The overall variances for a 1 ton bulk sample which is sub-sampled in three stages and the samples sent to the lab for replicate assay are shown in Table VI. The relative variances for the splitting stages and for the assay variance are taken from Tables V and IV.

In Table V the relative variances for a single measurement, the relative variances for replicate measurements and the overall relative variances for the whole process are presented.

The step that has the biggest relative variance for a single measurement is the assay in the lab (Relative variance 0.0168). The contribution of assay to the overall variance can be minimized by doing replicate assays.

Two scenarios are presented. In the first, duplicate 500 gm pulverized samples are sent to the lab for triplicate assay, giving 6 assays for each bulk sample. With this scenario, the CV% overall is 13.7% which is adequate.

In the second scenario, two 2.5 cm sub-samples are taken and two 0.3 cm sub-samples taken from each of these giving 4 sub-samples. These are each pulverized and two pulverized samples produced giving 8 samples submitted to the lab for duplicate assay. There would then be 16 individual assay results and the CV% overall would be 9%.

Sample	Number of trials	Mean grade g/t	Variance for single measurement	CV% single measurement
SARM53	300	4.056	0.0171	13.00%
Duplicate wastes	16	0.215	0.0165	12.80%

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Table V

Overall variance for a 1 ton bulk sample

Stage	Mass kgs	Particle size cms	Relative variance Tables III and IV	Scenario 1			Scenario 2		
				Number of replicates	Mean relative variance	CV%	Number of replicates	Mean relative variance	CV%
Bulk sample	1000	30	0.00216	1	0.00216	4.6	1	0.00216	4.6
Sub-sample 1	30	2.5	0.005999	1	0.005999	7.7	2	0.003	5.5
3 mm sub-sample	3	0.3	0.007199	1	0.007199	8.5	4	0.0018	4.2
Pulverized lab sample	0.5	0.0075	0.00108	2	0.00054	2.3	8	0.000135	1.2
Assay			0.0168	6	0.0028	5.3	16	0.00105	3.2
Overall					0.0187	13.7		0.0081	9.0

The scenario with single sub-samples and two pulverized samples assayed in triplicate would probably be used as a routine. The second scenario, with the replicate sub-samples would be used as an audit procedure and to confirm the variance models using 'Component of Variance Analysis'.

Discussion

Components of Variance

The statistical model is used to calculate the relative variances at the different particle sizes and sample masses. The converse of this is to use statistical data from the operation of the sampling plant to determine the contribution of each step to the overall variance. This is called a 'Components of Variance Analysis'. An application of this statistical technique is given for sampling of slurries in a flotation plant by Bartlett⁹.

When it is necessary to devise a sampling strategy, the first approach is to decide on the size of sub-samples and the particle size to which they should be crushed on the basis of the most suitable model available. Once data are available using the chosen strategy, a Components of Variance analysis could be performed to measure the variance that is actually being achieved at the different particle sizes. On the basis of the Components of Variance Analysis, the strategy can then be revised to include, for example, more replicate sub-samples at a particular particle size or to reduce the number of assays on the samples sent to the lab. The data gathered from the Components of Variance Analysis could be used to revise the sampling model. Hence there is a continuous process of improvement with better and better models being obtained.

This approach can be applied to many sampling situations. At a simple level, it would be advantageous to know the variance associated with the sampling and analysis of single drill cores. In this case, a series of drill cores would be divided into two portions. Each portion would be separately, crushed and ground and two sub-samples taken from each. These would be assayed in duplicate, giving eight assays in all. From the Components of Variance Analysis the variance of splitting the sample in two, the variance of crushing and splitting and the variance of assay can be determined.

Mass of samples

The mass of sample that is taken is incidental to the procedures used to take the sample. For the run-of-mine sampling presented in Table VI, the mass of the primary sample is between 600 kg and 4 tons. This mass comes about because the size of each individual increment is about 50 kg. The 50 kg increment is the result of having a particular sized cutter and taking between 13 and 26 increments per day. The variance from the composite sample would be different to the variance for a single 600 kg to 4 ton sample.

It is not appropriate therefore to have a formula relating the mass of sample to its variance in isolation to the methods used to obtain that mass.

Nature of the heterogeneity of gold ores

For many of the gold ores in the Witwatersrand, the gold is in a conglomerate. At the stope face, there are typically one or more layers of conglomerate with gold interstitial to quartz pebbles and with layers of barren quartz above and below. In mining, where minimum stope widths of 70 cm are the lowest that can be practically attained, the conglomerate and barren rock are broken out together. There are other sources of waste from the hanging and footwalls, from waste development and from mis-tramming of waste into reef ore passes. Therefore, particularly at the larger particle sizes one particle of ore is different to another because of varying amounts of conglomerate in the two particles and not due to the microscopic variations in the gold mineralogy. This situation is exacerbated with low grade gold ore. For this the gold is often in discrete lumps of conglomerate in waste rock and has come about because of misplacement due to incorrect tramming, or, the low grade ore had some reef in it but was not treated at the time it was mined because the grade was too low.

Accordingly, the heterogeneity of gold ores is mainly due to the macroscopic phenomena of different proportions of the conglomerate as a whole being in different particles.

Based on this argument, it appears that the microscopic mineralogical factors such as the shape factor, f , the granulometric factor, g , and the liberation factor, l , which are used in the original Gy equation, apart from being difficult or impossible to measure, would play only a minor role in the

Variance relationships between the masses, grades and particle sizes for gold ores

variance of samples with large particle sizes as found in the run-of-mine situation. It is therefore inappropriate, for gold ores from the Witwatersrand, to explain the variance, mass and particle size of samples in terms of the microscopic mineralogical factors.

As shown in this paper, the relationships can be measured by experiment using either the procedures suggested by François-Bongarçon or by the appropriate statistical procedures involving multiple linear regression or components of variance analysis.

Segregation of gold ores

In the process of reducing the particle size of gold ores by crushing, grinding and ultimately milling, there is a progressive liberation of gold particles. In mining operations, the finer fractions produced during blasting of the rock faces collect in footwall cracks underground and extensive sweeping campaigns have to be mounted to collect these fines.

For run-of-mine sampling, the fines are sometimes at the bottom of the belts conveying the ore. Biases in sampling could therefore come about because the fines are not collected when the arm collecting the sample passes over the belt. However, this is not usually a major problem as the quantity of fines is usually small and assays of these fines do not indicate the presence of free gold. However, as the ore is crushed and split in the sample preparation stages free gold is liberated. The separation of gold becomes progressively greater as the rock is crushed to finer sizes and more gold is liberated. This tendency for enrichment is illustrated in Table I where it can be seen that the grade of ore in the particles of 3.14 cm is 0.17 g/t whereas at 38 microns the grade is 1.85 g/t.

By gravity separation, these richer finer fragments tend to concentrate at the bottom of chutes and launders. If proper attention is not paid to this, substantial biases in sampling can be found, not only in the primary sampling but also during sample preparation. It has been found, for example, that the use of incorrectly designed turning tube dividers for sampling ore at a size of 3 mm or 6mm results in substantial biases because the heavy gold particles go to the bottom of the rotating sample tube. For turning tube dividers that were used at one particular mine, the cutter on the side of the divider was positioned such that this material at the bottom was not representatively sampled and substantial biases in the run-of-mine sampling were found.

Gravity separation plays a role also in the sampling of pulp streams in concentrators and gold plants. This is discussed in the companion paper to this one.

Conclusions

- ▶ A model to relate the variance of samples of Witwatersrand gold ores to the maximum particle size, the mass and grade of the samples is presented.
- ▶ The model was determined using statistical analysis of data available from crushing experiments on low grade ore. There was a close agreement between the

relationships between variance, mass and particle size of sample determined by statistical analysis and the relationships determined experimentally using the methods originally proposed by François-Bongarçon.

- ▶ A combined model using the data from the crushing experiment and data on other Witwatersrand gold ores extended the range of the models and enabled the model to be used with varying grades from 0.5 g/t to 60 g/t gold.
- ▶ The variance, mass, grade and particle size relationships for gold ores from the Witwatersrand are determined by the macroscopic distribution of conglomerate reef particles in a host matrix of barren quartz. Therefore, the microscopic features of the distribution of discrete gold particles as estimated from various factors would only play a minor role in the relationships.
- ▶ An example is given of the use of the model to estimate the variances that would be obtained at different stages in the sampling and sub-sampling process. These variances would be used to establish a sampling protocol.
- ▶ Having established the protocol, components of variance analyses would be conducted to assess how closely the model relates to the actual situation. In this way there could be a continuous improvement of the sampling.

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Appendix

Model log (variance *mass) versus log size log grade for low grade gold ores

The GLM Procedure

Number of observations 58

Dependent Variable: log - var - mass

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	2	185.2841609	92.6420805	37.05	<.0001
Error	55	137.5354514	2.5006446		
Corrected Total	57	322.8196123			

R-Square	Coeff Var	Root MSE	Log var mass Mean
0.573956	52.18544	1.581343	3.030237

Source	DF	Type I SS	Mean Square	F Value	Pr > F
Log_size	1	182.3259871	182.3259871	72.91	<.0001
Log_meanau	1	2.9581738	2.9581738	1.18	0.2815

Source	DF	Type III SS	Mean Square	F Value	Pr > F
Log_size	1	99.06714249	99.06714249	39.62	<.0001
Log_meanau	1	2.95817381	2.95817381	1.18	0.2815

Parameter	Estimate	Standard Error	t Value	Pr > t
Intercept	5.559478809	0.74037135	7.51	<.0001
Log_size	1.047317015	0.16639464	6.29	<.0001
Log_meanau	0.404131811	0.37156704	1.09	0.2815