A chance-constrained programming approach for open pit long-term production scheduling in stochastic environments

by J. Golamnejad*, M. Osanloo*, and B. Karimi*

Synopsis

This paper attempts to model long-term production scheduling problems by chance constrained binary integer programming in a stochastic environment. This stochastic model is set up to account for ore block grade uncertainty. The probability distribution function of grade in each block is used as a stochastic input to the optimization model. This distribution function in each block should be determined using a Geostatistical Simulation approach. The deterministic equivalents of these chance constraints are then achieved, which are in the form of nonlinear in binary variables. A confidence level at which it is desired that the uncertain constraints holds, is specified in each scheduling period. Rather than the previous risk-based model, this formulation will yield schedules with a high chance of achieving planned production targets while maximizes the expectation of net present value and minimizing the variance function simultaneously. Using this method, the grade uncertainty is integrated explicitly into the optimization process.

Introduction

A long-term production scheduling optimization process is a major step in mine planning. It aims to maximize the net present value of the total profits from the production process while satisfying all the operational constraints, such as mining slope, grade blending, ore production, mining capacity, etc. during each scheduling period with a predetermined high degree of probability. Optimization and mathematical formulation have been exploited to solve long-term production scheduling problems since the 1960s. There are two mathematical optimization approaches to solving these kind of problems: deterministic and uncertainty-based approaches. In deterministic models, all inputs are assumed to have fixed known real values.

However, the assumption of inputs certainty is not always realistic. In reality, some of the data—ore grades, future product demand, future product price and production costs—can vary within certain limits (for our discussion, randomly) and we have to make our decision on the production plan before knowing the exact values of those data. Any of the deterministic methods are unable to deal with uncertainty in a quantitative manner. This will result in generating infeasible schedules in terms of production requirements.

The uncertainty-based approach to optimizing open pit mine design and scheduling was developed in the 1990s. Ravenscroft discussed risk analysis in mine production scheduling[1]. This method can show only the impact of grade uncertainty on production scheduling using the alternative scenarios of the orebody, which are provided by conditional simulation. Dowd proposed a framework for risk assessment in open pit mines[2]. He also considered some other random variables such as commodity price, mining costs, processing costs, etc. Denby and Schofield proposed an algorithm, which considers ore grade variance in open pit design and scheduling using the Genetic algorithm[3]. They used a multi-objective optimization method: maximizing value and minimizing risk. Dimitrakopoulos and Ramazan discussed an LP approach that considered grade uncertainty, equipment access and mobility constraints[4]. This formulation was based on expected ore block grades and the probabilities of different element grades being above required cut-offs, both derived from simulated orebody models. Gody and Dimitrakopoulos presented an algorithm, which addresses the generation of an optimal condition under uncertainty[5]. At first, they generated production scheduling on each simulated orebody and then they combined the mining sequences to produce a single schedule that minimizes the chance of deviating from the

* *Amirkabir University of Technology, Tehran, Iran.*
© *The South African Institute of Mining and Metallurgy, 2006.* *SA ISSN 0038–223X/3.00 + 0.00. Paper received Jan. 2005; revised paper received Jun. 2005.*

production target. This was done using the Simulated Annealing Meta-Heuristic method. Ramazan and Dimitralopoulos suggested an MIP model that accommodates grade uncertainty[6]. In this method, after obtaining simulated orebody models, scheduling patterns on each model is generated using traditional MIP formulation (with the objective of NPV maximization). Then the excavation probability of each blocks in a given time period is calculated. The block with probability between zero and one, are considered in a new optimization model.

It has been shown that these risk-based models are unable to make explicit the integration of ore grade uncertainty and also present an optimal solution under uncertainty without conducting repeated boring traditional optimization on a simulated orebody.

This paper introduces the chance constrained binary integer programming model that explicitly and effectively incorporates grade uncertainty into optimization of long-term production scheduling in open pit mines. This model generates the schedules that maximize net present value of the total revenue and simultaneously, minimizes the risk of the schedules, which are originated from ore grade uncertainty, and also maintains a predetermined reliability with respect to satisfying probabilistic constraints. Also, the issues of equipment access and mobility constraints are considered in this model.

Uncertainty in open pit optimization

Long-term production scheduling is a challenging task in the theory of surface mining. The most important aspects of the optimum long-term production scheduling scenario are:

- It determines the distribution of cash flow over life of mine. This optimal scenario specifies how an orebody should be mined economically in such a way that several requirements are satisfied in each scheduling period
- Long-term plans are used as a basis for implementing cut-off grade strategy, and short-term and medium-term production scheduling; because of this, the optimality of the mentioned issues strongly depends on the optimality of the long-term production scheduling solution.

This optimal scenario is sensitive to the uncertainties concerned with the inputs to the optimization model. Dimitrakopoulos classified the uncertainties of mining projects as follows[7]:

- Uncertainty of the orebody model and related *in situ* grade variability and material type distribution
- Uncertainty of technical mining specifications such as slope constraints, excavation capacities, etc.
- Uncertainty of economic issues including capital and operating costs and commodity prices.

These uncertainties originate from the nature of the variables (such as the commodity price) and also the cost of the obtaining information about them (such as geological uncertainty).

Among these uncertainties, the uncertainty related to orebody model and *in situ* grade variability is a major contributor causing expectations not to be met in the early stages of a project. Valee reported that in the first year of operation after start-up, 60% of surveyed mines had an average rate of production less than 70% of designed capacity[8]. Geological uncertainty is identified as a major contributor to these shortfalls. Indeed geological risk—which originates from geological uncertainties—cannot be eliminated; consequently, the best solution is to quantify the grade uncertainties, reduce these uncertainties so far as invest permits and finally, manage the residual risk of grade uncertainty.

Kriging is a geostatistical method which estimates the ore block grade so that the mean squared error is minimized; therefore, the variance of value estimated by kriging is smaller than the real but unknown variance. This smoothing of true variability of the grade leads to overestimation of low grades and underestimation of high grades. Additionally, smoothing is a function of data density and configuration: areas of greater density will show more local variability, while areas having sparse data will be more uniform[9]. Hence, production schedules that are based on Kriging cannot account for probable deviation in production targets and obtained plans will provide only erroneous conclusions. Multiple indicator kriging works on a probabilistic basis to define the distribution of grade of samples within each search window, providing a discrete approximation to the conditional cumulative distribution function for each block[10]. Although the probabilistic estimates produced by the multiple indicator kriging method often reduce smoothing effect, the local grade variability may still be incorrectly characterized.

The best way to quantify grade uncertainty is conditional simulation. Conditional simulation is a generalization of the Monte Carlo type simulation approach, which considers three-dimensional spatial correlation[11,12]. It produces independent equally probable images of *in situ* orebody grades, which have the following characteristics[13]:

- At the sampled location, the simulated values of each variable are the same as the measured values of those variables
- All the simulated values of a given variable have the same spatial relationships as observed in the data value
- All the simulated values of any pair of variables have the same spatial interrelationships as observed in the data values
- The histograms of the simulated values of all variables are the same as those observed for the data images.

Each simulation run produces an image or a realization of the deposits that correctly reflect the statistical and spatial variability of the real data. Performing several independent simulations is required to assess the impact of local variability.

Reducing the uncertainty of the ore block grade can be achieved by spending more money to increase drill hole density. This infill drilling should be conducted preferably in the high grade zones associated with a high level of uncertainty. These zones can be identified by using the results of conditional simulation.

Finally, mine planning and scheduling should be managed in such a way that the residual risk of grade uncertainty is minimized in design procedure. The keynote in this regard is extraction of high grade and certain low zones in earlier production periods and leaving more uncertain zones for the later periods such that all mining constraints are satisfied with a high level of confidence. In the following sections we deal with this important point using stochastic programming.

Stochastic version of the long-term production scheduling model

The stochastic programming based production scheduling method can be summarized as follows:

- To develop an economic block model
- To find ultimate pit limits using one of the optimum methods such as Learchs and Grossman's[14,15] and maximum flow network methods[16,17]
- To develop a stochastic integer programming model for optimal selection of the blocks in each period on the basis of specified criteria
- To obtain the deterministic equivalent on the mentioned stochastic model
- To solve the deterministic form of the stochastic model for long-term production scheduling problems using efficient zero-one integer programming methods.

Formulation of the long-term production scheduling problem

Symbols

t: Planning period index, t=1, 2,..., T

T: Total number of scheduling periods in the planning horizon

I: Total number of blocks into which mineral deposits have been divided, such as each block has the potential to be mined and processed in one scheduling period

K: Total number of waste blocks

$I+K$: Total number of blocks to be scheduled

i: Block identification number which is i=1,2,...,I for mineralized blocks and i=I+1, I+2,..., I+K for waste/overburden blocks

d: Discount rate in each period

x_i^t: A binary decision variable which is equal to 1 if the block i is to be mined in period t and 0 otherwise

P^t: Unit selling price of the metal in period t

SP^t: Unit selling cost of the metal in period t

P_c^t: Unit processing cost of the ore in period t

M_{co}^t: Unit mining cost of the mineralized material in period t

M_{cw}^t: Unit mining cost of the waste/overburden material in period t

$\tilde{g}_i$: Block grade which is a random variable (i=1, 2, ...,I)

$E(\tilde{g}_i)$: Expected value of $\tilde{g}_i$

$\mathrm{Var}(\tilde{g}_i)$: Variance of $\tilde{g}_i$

$\mathrm{Cov}(\tilde{g}_i,\tilde{g}_j)$: Covariance between $\tilde{g}_i$ and $\tilde{g}_j$.

C_i^t: Net present value to be generated by mining block i in period t. Where there is only one commodity in the mining system and block i is categorized as an ore block, C_i^t can be calculated as follows:

$$C_i^t = \frac{1}{(1+d)^t}\begin{Bmatrix}\left[\left(P^t - SP^t\right).\tilde{g}_i.R - P_c^t\right].\\ TB_i - \left(M_{co}^t.TB_i\right)\end{Bmatrix} \quad [1]$$

For $i = 1,2,...,I$

But if mineralized block i is classified as a waste block, C_i^t should be calculated using following equation:

$$C_i^t = \frac{1}{(1+d)^t}\left\{-\left(M_{co}^t.TB_i\right)\right\} \quad [2]$$

And for no mineralized waste block we have:

$$C_i^t = \frac{1}{(1+d)^t}\left\{-\left(M_{cw}^t.TB_i\right)\right\} \quad [3]$$

For $i = I+1,\ I+2,...,I+K$

V: The IxI covariance matrix whose i-th diagonal element represents $Var(\tilde{g}_i)$ and the element of i-th row and j-th column represent $Cov(\tilde{g}_i,\ \tilde{g}_j)$. Thus we have:

$$V = \begin{bmatrix} Var(\tilde{g}_1) & Cov(\tilde{g}_1,\tilde{g}_2) & Cov(\tilde{g}_1,\tilde{g}_3) & \cdots & Cov(\tilde{g}_1,\tilde{g}_I)\\ Cov(\tilde{g}_2,\tilde{g}_1) & Var(\tilde{g}_2) & Cov(\tilde{g}_2,\tilde{g}_3) & \cdots & Cov(\tilde{g}_2,\tilde{g}_I)\\ \vdots & \vdots & \vdots & \cdots & \vdots\\ Cov(\tilde{g}_I,\tilde{g}_1) & Cov(\tilde{g}_I,\tilde{g}_2) & Cov(\tilde{g}_I,\tilde{g}_3) & \cdots & Var(\tilde{g}_I)\end{bmatrix} \quad [4]$$

R: Total metal recovery

TB_i: The total ore/waste/overburden tonnages in block i

$G_{\max}^t$: The upper bound average grade of material sent to the mill in period t

$G_{\min}^t$: The lower bound average grade of material sent to the mill in period t

$PC_{\max}^t$: The upper bound total tons of ore processed in period t

$PC_{\min}^t$: The lower bound total tons of ore processed in period t

$MC_{\max}^t$: The upper bound total amount of material (waste and ore) to be mined in period t

$MC_{\min}^t$: The lower bound total amount of material (waste and ore) to be mined in period t

b: The index of a block considered for extraction in period t

m: The total number of blocks overlaying block b

l: The counter for the m overlaying blocks

q: The total number of blocks surrounding block b

o: The counter for the q surrounding blocks

α_t: The least probability of fulfilling the demand in period t (confidence level). $\propto_t$, in fact, represents the acceptable risk level for not fulfilling the demands in period t

L_O: The allowable lower limit of the rate of extraction
U_o: The allowable upper limit of the rate of extraction
Pr: The probability operator

Formulation of the multi-period long-term production scheduling model

In light of the definition described above, the multi-period production scheduling model will be formulated as follows:

Objective function

$$\text{Maximize } Z = \sum_{t=1}^{T}\sum_{i=1}^{I+K} C_i^t.x_i^t \quad [5]$$

This objective function is subjected to the following constraints:

- *Grade blending constraints*—the average grade of the mineralized material sent to the mill has to be more than a lower bound (G_{min}^t) and less than an upper bound (G_{max}^t) in each period:

$$\frac{\sum_{i=1}^{I}\tilde{g}_i.TB_i.x_i^t}{\sum_{i=1}^{I}TB_i.x_i^t} \le G_{\max}^t \quad \text{For } t = 1,2,...,T \quad [6]$$

$$\frac{\sum_{i=1}^{I}\tilde{g}_i.TB_i.x_i^t}{\sum_{i=1}^{I}TB_i.x_i^t} \ge G_{\min}^t \quad \text{For } t = 1,2,...,T \quad [7]$$

- *Processing capacity constraint*—the total tons of ore processed in each period should be more than a lower bound (PC_{min}^t) and less than an upper bound (PC_{max}^t):

$$\sum_{i=1}^{I} TB_i.x_i^t \le PC_{\max}^t \quad \text{For } t = 1,2,...,T \quad [8]$$

$$\sum_{i=1}^{I} TB_i.x_i^t \ge PC_{\min}^t \quad \text{For } t = 1,2,...,T \quad [9]$$

- *Mining capacity constraints*—The total tones of waste and ore to be mined should be more than a lower bound (MC_{min}^t) and less than an upper bound (MC_{max}^t):

$$\sum_{i=1}^{I+K} TB_i.x_i^t \le MC_{\max}^t \quad \text{For } t = 1,2,...,T \quad [10]$$

$$\sum_{i=1}^{I+K} TB_i.x_i^t \ge MC_{\min}^t \quad \text{For } t = 1,2,...,T \quad [11]$$

- *Reserve constraints*—reserve constraints insure that any block in the model is mined only once:

$$\sum_{t=1}^{T} x_i^t = 1 \quad \text{For } i = 1,2,...,I,\ I+1,..,I+K \quad [12]$$

- *Slope constraints*—these constraints ensure that all blocks which directly restrict the mining of a given block *b* must be completely mined out before the mining of block *b* starts. To represent the restricting blocks, a cone template can be made that either contains 5 (Figure 1-a) or 9 blocks (Figure 1-b) above block *b*. In this study a cone template with 9 blocks will be used. There are two methods for implementing these constraints[18]:

Using one constraint for each block per period:

$$m.x_b^t - \sum_{l=1}^{m}\sum_{r=1}^{t} x_1^r \le 0 \quad \text{For } t = 1,2,...,T \quad [13]$$
$$\text{and } b = 1,2,...,I,...,I+K$$

Using *m* constraints for each block per period:

$$x_b^t - \sum_{r=1}^{t} x_1^r \le 0 \quad \text{For } t = 1,2,...,T, \quad [14]$$
$$b = 1,2,...,I,...,I+K \text{ and } 1 = 1,2,....,m$$

Ramazan and Dimitrakopoulos showed that in large size models it may be better to use Equation [14][19]. Although using Equation [14] will increase the size of the model, in some cases it may significantly decrease the computing time by decreasing the search space.

- *Production tolerance constraints*—the amount of changes in the level of production from one scheduling period to the next are usually limited by the amount of changes permitted in the transportation system of the pit, water pumping system and manpower employment stabilization policy of the company. These constraints can be formulated as follows[20]:

$$L_o\left(\sum_{i=1}^{I+K} TB_i.x_i^{t-1}\right) \le \sum_{i=1}^{I+K} TB_i.x_i^t \le U_o\left(\sum_{i=1}^{I+K} TB_i.x_i^{t-1}\right) \quad [15]$$
$$\text{For } t = 2,3,...,T$$

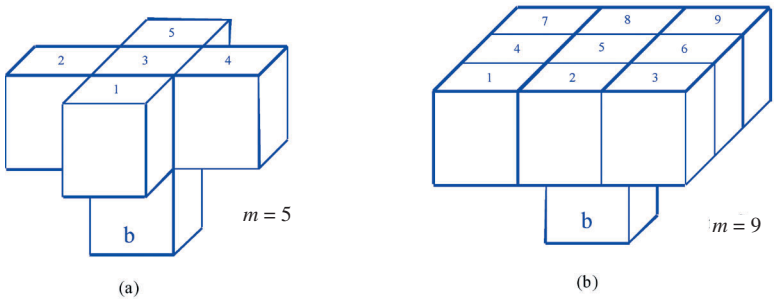

Figure 1—Two types of cone templates to model the slope constraints

- *Equipment access and mobility constraints*—these constraints ensure that there is sufficient access to equipments for mining a given block and they also prevent the spreading of the scheduling pattern over each period. The latter will reduce the equipment movement in a given period; hence, introducing these constraints leads to creating feasible and more practical schedules. If block size, equipment size, width of ramp and other operational constraints dictate that block b should be mined together with the q surrounding blocks, then these constraints can be written as:

$$q.x_b^t - \sum_{o=1}^{q} x_o^t \le 0 \quad \text{For } t = 1,2,...,T \qquad [16]$$

and $b = 1,2,..., I..., I+K$

For example q in Figure (2-a) and (2-b) are equal to 8 and 24, respectively.

Stochastic formulation of the multi-period long-term production scheduling model

As mentioned before, an ore block's grade ($\tilde{g}_i$) is not known with certainty at the beginning of each planning period. We have only statistical information about the random grades; therefore, the objective function and constraints 6 and 7 contain random parameters. The main difficulty of such models is due to optimal decisions that have to be taken prior to the observation of random parameters.

There are several approaches to handling the uncertainty of this problem. The two most common approaches are the two-stage stochastic programming method and the chance-constrained stochastic programming method. In the first one, the decision maker takes some action in the first stage, after which a random event occurs affecting the outcome of the first stage decisions. A recourse decision can then be made in the second stage that compensates for any bad effects that might have been experienced as a result of the first-stage decision. Constraints violation caused by unexpected random effects can be balanced afterwards by some compensating decisions in the second stage. As long as the costs of compensating decisions are known, these may be considered as a penalization for constraint violation. In this method the stochastic constraints have to hold almost surely, i.e., they are to be satisfied with probability of 1[21].

In many applications, however, compensations simply do not exist or cannot be modelled as costs in any reasonable way. In such circumstances, one would rather insist on decisions guaranteeing feasibility 'as much as' possible. This loose term refers once more to the fact that constraints violations can almost never be avoided because of extreme events, i.e., a low percentage realization of the random parameters leads to constraints violation under these fixed decisions. This approach is called chance-constrained programming.

Chance-constrained programming was formulated originally by Charnes, Cooper and Symonds[22] and Charnes and Cooper[23] and then developed and applied by Charnes and Cooper[24].

In this section the chance-constrained programming approach is exploited to handle block grade uncertainty for the proposed binary integer model.

As stated before, among the model constraints only constraints 6 and 7 contain random parameters. The generic way to express such constraints is:

$$Pr\left[\frac{\sum_{i=1}^{I} \tilde{g}_i.TB_i.x_i^t}{\sum_{i=1}^{I} TB_i.x_i^t} \le G_{\max}^t\right] \ge \alpha_t \qquad [17]$$

For $t = 1,2,...,T$

$$Pr\left[\frac{\sum_{i=1}^{I} \tilde{g}_i.TB_i.x_i^t}{\sum_{i=1}^{I} TB_i.x_i^t} \ge G_{\min}^t\right] \ge \alpha_t \qquad [18]$$

For $t = 1,2,...,T$

The value of $\alpha_t \in [0,1]$ is called the probability level, and it is chosen by the decision maker in order to model the safety requirements. It should be noted that higher values of α_t results in fewer feasible solutions x_i^t in Constraints [17] and [18], and hence yields an optimal solution at a lower NPV. Henrion stated that usually α_t can be increased over quite a wide range without greatly affecting the optimal value of some problem, until it closely approaches 1 and then a strong decrease of NPV becomes evident[25].

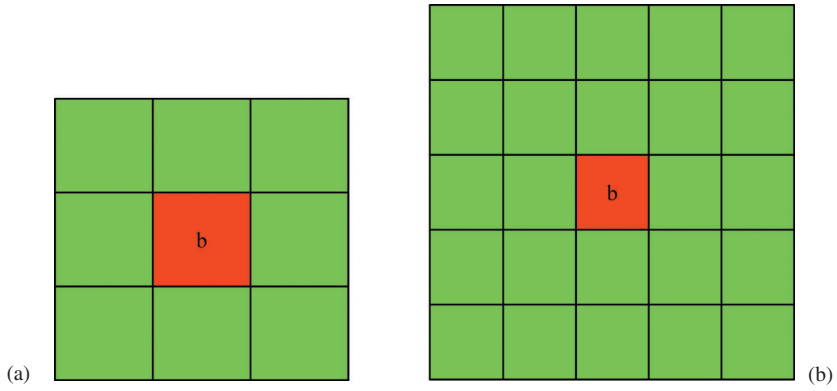

Figure 2—Block configurations around block *i*

With regard to the stochastic nature of the objective function and also Constraints [17] and [18], above problem is not well defined; consequently, a revision of the modelling process is necessary, leading to so-called deterministic equivalents for the stochastic programming model, i.e. one which no longer contains any probabilistic element.

Deterministic equivalent of chance-constrained binary integer programming model

In this section the chance-constrained binary integer programming model of the long-term production scheduling problem is reduced to its deterministic equivalent.

- *Objective function*—as is clear from Equation [5] due to the stochastic nature of C_i^t, Z is also a random variable with the expected value and variance of:

$$E(Z)=\sum_{t=1}^{T}\frac{1}{(1+d)^t}\left\{\sum_{i=1}^{I}\begin{bmatrix}[(P^t-SP^t).E(\tilde{g}_i).R-P_c^t].\\ TB_i.x_i^t-(M_{co}^t.TB_i).x_i^t\end{bmatrix}-\sum_{i=I+1}^{I+K}(M_{cw}^t.TB_i).x_i^t\right\} \quad [19]$$

$$Var(Z)=\frac{1}{(1+d)^{2t}}\left\{\sum_{t=1}^{T}\sum_{i=1}^{I}\left[(P^t-SP^t).R.TB_i.x_i^t\right]^2 Var(\tilde{g}_i)+\sum_{t=1}^{T}\sum_{i=1}^{I}\sum_{j=1}^{I}\left[(P^t-SP^t).R.TB_i\right]\times\left[(P^t-SP^t).R.TB_j\right].x_i^t.x_j^t.Cov(\tilde{g}_i,\tilde{g}_i)\right\}i\neq j \quad [20]$$

In Equation [19] if mineralized block i (i=1, 2,...,I) is classified as a waste block, then we have:

$$E\left(C_i^t.x_i^t\right)=\frac{1}{(1+d)^t}\left\{-(M_{co}^t.TB_i)\right\}.x_i^t \quad [21]$$

$$Var\left(C_i^t.x_i^t\right)=0 \quad [22]$$

Several classes of objective function can be identified, which result in different solutions. These include[24]:

- Expected value optimization objective
- A minimum variance objective
- A maximum probability model.

We set-up our objective for the maximization of expected value of net present value and the minimization of standard deviation simultaneously; as a result, the objective function of open pit long-term production scheduling can be written as:

$$\text{Maximize } Z=k_1.E(Z)-k_2.\sqrt{Var(Z)} \quad [23]$$

Where k_1 and k_2 are non-negative coefficients and show the relative importance of the maximization of expected value and minimization of standard deviation of net present value. If k_1=k_2 then these two objectives are of the same importance for the decision maker. Thus the objective function can be re-written as follows:

$$\text{Maximize } Z=k_1.\sum_{t=1}^{T}\frac{1}{(1+d)^t}\left\{\sum_{i=1}^{I}\begin{bmatrix}[(P^t-SP^t).E(\tilde{g}_i).R-P_c^t].TB_i.x_i^t-\\ (M_{co}^t.TB_i).x_i^t-\sum_{i=I+1}^{I+K}(M_{cw}^t.TB_i).x_i^t\end{bmatrix}\right\}$$
$$-k_2\left[\frac{1}{(1+d)^{2t}}\{\sum_{t=1}^{T}\sum_{i=1}^{I}\left[\begin{matrix}(P^t-SP^t).\\ R.TB_i.x_i^t\end{matrix}\right]^2.Var(\tilde{g}_i)+\sum_{t=1}^{T}\sum_{i=1}^{I}\sum_{j=1}^{I}\left[(P^t-SP^t).R.TB_i\right]\times\left[(P^t-SP^t).R.TB_j\right].x_i^t.x_j^t.Cov(\tilde{g}_i,\tilde{g}_j)\}\right]^{1/2} i\neq j \quad [24]$$

Let define:

$$q_i^t=\left[(P^t-SP^t).E(\tilde{g}_i).R-P_c^t-M_{co}^t\right]TB_i.\frac{1}{(1+d)^t} \quad [25]$$

$$S_i^t\frac{1}{(1+d)^t}\left[M_{cw}^t.TB_i\right] \quad [26]$$

$$A_i^t=\frac{1}{(1+d)^{2t}}\left[(P^t-SP^t).R.TB_i\right]^2.Var(\tilde{g}_i) \quad [27]$$

$$W_{ij}^t=\frac{1}{(1+d)^t}\left[(P^t-SP^t).R.TB_i\right]\times\left[(P^t-SP^t).R.TB_j\right]Cov(\tilde{g}_i,\tilde{g}_j) \quad [28]$$

Thus, the objective function can be simplified as follows:

$$\text{Maximize } Z=k_1.\left[\sum_{t=1}^{T}\sum_{i=1}^{I}q_i^t.x_i^t-\sum_{t=1}^{T}\sum_{i=I+1}^{I+K}S_i^t.x_i^t\right]-k_2\sqrt{\sum_{t=1}^{T}\sum_{i=1}^{I}\left[A_i^t\left(x_i^t\right)^2+\sum_{j=1}^{I}W_{ij}^t.x_i^t.x_j^t\right]}\ i\neq j \quad [29]$$

In zero-one integer programming we have $(x_i^t)^2=x_i^t$; because of this, the final shape of the objective function has the form of:

$$\text{Maximize } Z=k_1.\left[\sum_{t=1}^{T}\sum_{i=1}^{I}q_i^t.x_i^t-\sum_{t=1}^{T}\sum_{i=I+1}^{I+K}S_i^t.x_i^t\right]-k_2\sqrt{\sum_{t=1}^{T}\sum_{i=1}^{I}\left[A_i^t.x_i^t+\sum_{j=1}^{I}W_{ij}^t.x_i^t.x_j^t\right]}\ i\neq j \quad [30]$$

- *Stochastic constraints*—in this section we will obtain the deterministic equivalents of stochastic constraints. In Equations [17] and [18] let us define:

$$d_t=\frac{\sum_{i=1}^{I}\tilde{g}_i.TB_i.x_i^t}{\sum_{i=1}^{I}TB_i.x_i^t} \quad [31]$$

Thus, Equation [17] can be rewritten as:

$$Pr\left[d_t \le G^t_{\max}\right] \ge \alpha_t \quad \text{For } t = 1,2,\ldots,T \qquad [32]$$

As mentioned before d_t,is the average grade of blocks to be scheduled in period t which is a random variable. According to the Central Limit Theorem, the distribution of d_t can be approximated by a normal distribution function with the following mean and variance:

$$E(d_t) = \frac{\sum_{i=1}^{I} E(\tilde{g}_i).TB_i.x_i^t}{\sum_{i=1}^{I} TB_i.x_i^t} \qquad [33]$$

$$Var(d_t) = \frac{\sum_{i=1}^{I} TB_i^2.Var(\tilde{g}_i).x_i^t + \sum_{i=1}^{I}\sum_{j=1}^{I}\left(TB_i.x_i^t\right)\left(TB_j.x_j^t\right).Cov\left(\tilde{g}_i,\tilde{g}_j\right)}{\sum_{i=1}^{I} TB_i.x_i^t} \qquad [34]$$

If we minus $E(d_t)$ from both sides of Equation [32], and then divide by $\sqrt{Var(d_t)}$, Equation [32] can be re-written as:

$$Pr\left[\frac{d_t - E(d_t)}{\sqrt{Var(d_t)}} \le \frac{G^t_{\max} - E(d_t)}{\sqrt{Var(d_t)}}\right] \ge \alpha_t \qquad [35]$$

By defining, then D_t $\frac{d_t - E(d_t)}{\sqrt{Var(d_t)}}$, then D_t has a standard normal distribution function which has a zero mean and unit standard deviation. A value of $K_{\alpha t}$ can then be determined from the area under the normal curve such that:

$$Pr(D_t \le K_{\alpha t}) = \int_{-\infty}^{K_{\alpha t}} \frac{1}{2\pi}.\exp\left(-\frac{x^2}{2}\right)dx = \alpha_t \qquad [36]$$

Thus, combining Equations [35] and [36] yields:

$$\frac{G^t_{\max} - E(d_t)}{\sqrt{Var(d_t)}} \ge K_{\alpha t} \Rightarrow E(d_t) + K_{\alpha t}\sqrt{Var(d_t)} \le G^t_{\max} \qquad [37]$$

With the combination of Equations [33], [34] and [37] the deterministic equivalent form of Constraints [17] can be stated as follows:

$$\sum_{i=1}^{I}\left(E(\tilde{g}_i) - G^t_{\max}\right).TB_i.x_i^t + K_{\alpha t}.\sqrt{\sum_{i=1}^{I}(TB_i)^2.x_i^t.Var(\tilde{g}_i) + \sum_{i=1}^{I}\sum_{j=1}^{I}\left(TB_i.x_i^t\right)\left(TB_j.x_j^t\right).Cov\left(\tilde{g}_i,\tilde{g}_j\right)} \le 0 \qquad [38]$$

Usually we take $\alpha_t \ge 0.5$; then $K_{at} \ge 0$.

Similarly, the deterministic equivalent of Equation [18] is of the form of:

$$\sum_{i=1}^{I}\left(E(\tilde{g}_i) - G^t_{\min}\right).TB_i.x_i^t + K'_{\alpha t}.\sqrt{\sum_{i=1}^{I}(TB_i)^2.x_i^t.Var(\tilde{g}_i) + \sum_{i=1}^{I}\sum_{j=1}^{I}\left(TB_i.x_i^t\right)\left(TB_j.x_j^t\right).Cov\left(\tilde{g}_i,\tilde{g}_j\right)} \ge 0 \qquad [39]$$

Where:

$$Pr\left(D_t \le K'_{\alpha t}\right) = \int_{-\infty}^{K'_{\alpha t}} \frac{1}{2\pi}.exp\left(-\frac{x^2}{2}\right)dx = 1 - \alpha_t \qquad [40]$$

Since we take $\alpha_t \ge 0.5$; then $1\text{-}\alpha_t \le 0.5$ that results in $K'_{\alpha t} \le 0$.

As is clear from Equations [38] and [39], the deterministic equivalents of probabilistic constraints are nonlinear. However, other constraints are themselves deterministic and remain unchanged.

Final shape of open pit long-term production scheduling model

The final shape of the open pit long-term production scheduling problem can be expressed as a nonlinear binary integer programming model and is summarized in Table I.

The above formulation has $(I+K).T$ binary integer variables and either $7T+(I+K)(2T+1)$-1 (when one slope constraint per block per period is used) or $7T+(I+K)(T.m+T+1)$-1 (when m slope constraints per block per period is used) linear constraints. Typically T is around 10, and $I+K$ is 70 000 for a small pit and over 1 000 000 for a large pit; consequently, the minimum and maximum sizes of such a problem are shown in Table II.

With regard to the size of the open pit mine scheduling problem, the model cannot be solved using exact or even heuristic methods; therefore, meta-heuristics like the genetic algorithm, Tabu search, etc. is suggested for solving the model.

Mineralized block classification

One of challenges in mine planning and scheduling is mineralized block classification. This categorization determines the block destination and, therefore, the block economic value. For example, in a gold mine, a mineralized block can be sent to mill, dump leach, heap leach or waste dump. Every destination leads to special recovery, costs and, consequently, a special block revenue. Misclassification of a block results in an incorrect block value calculation. In this case, economic loss function is used to differentiate between ore and waste block. This function was first introduced by Issaks in 1990[26]. The actual economic loss associated with each type of mining category n (n=1, 2,...,N and ordered from lowest grade to highest grade) is the potential value less the recovered value, which can be calculated using the following equation:

$$L_n(i) = \left[(P - SP) \times g_i \times R_n - M_{cn} - P_{cn}\right] - \left[(P - SP) \times g_i \times R_{n'} - M_{cn'} - P_{cn'}\right] \qquad [41]$$

Table I

The final shape of an open pit long-term production scheduling problem

Objective Function:

$$\text{Maximize } Z = k_1\left[\sum_{t=1}^{T}\sum_{i=1}^{I} q_i^t.x_i^t - \sum_{t=1}^{T}\sum_{i=I+1}^{I+K} S_i^t.x_i^t\right] - k_2\sqrt{\sum_{t=1}^{T}\sum_{i=1}^{I}\left[A_i^t.x_i^t + \sum_{j=1}^{I} W_{ij}^t.x_i^t.x_j^t\right]}$$

Grade blending constraints:

$$\sum_{i=1}^{I}\left(E(\tilde{g}_i) - G_{max}^t\right)TB_i.x_i^t + K_{\alpha t}.\sqrt{\sum_{i=1}^{I}(TB_i)^2.x_i^t.Var(\tilde{g}_i) + \sum_{i=1}^{I}\sum_{j=1}^{I}\left(TB_i.x_i^t\right)\left(TB_j.x_j^t\right)Cov(\tilde{g}_i,\tilde{g}_j)} \le 0$$

For $t=1,2,\ldots,T$

$$\sum_{i=1}^{I}\left(E(\tilde{g}_i) - G_{min}^t\right)TB_i.x_i^t + K'_{\alpha t}.\sqrt{\sum_{i=1}^{I}(TB_i)^2.x_i^t.Var(\tilde{g}_i) + \sum_{i=1}^{I}\sum_{j=1}^{I}\left(TB_i.x_i^t\right)\left(TB_j.x_j^t\right)Cov(\tilde{g}_i,\tilde{g}_j)} \ge 0$$

For $t=1,2,\ldots,T$

Processing Capacity Constraints:

$$\sum_{i=1}^{I} TB_i.x_i^t \le PC_{max}^t \qquad \text{For } t=1,2,\ldots,T$$

$$\sum_{i=1}^{I} TB_i.x_i^t \ge PC_{min}^t \qquad \text{For } t=1,2,\ldots,T$$

Mining Capacity Constraints:

$$\sum_{i=1}^{I+K} TB_i.x_i^t \le MC_{max}^t \qquad \text{For } t=1,2,\ldots,T$$

$$\sum_{i=1}^{I+K} TB_i.x_i^t \ge MC_{min}^t \qquad \text{For } t=1,2,\ldots,T$$

Reserve Constraints:

$$\sum_{t=1}^{T} x_i^t = 1 \qquad \text{For } i=1,2,\ldots,I,I+1,\ldots,I+K$$

Slope Constraints:

$$m.x_b^t - \sum_{l=1}^{m}\sum_{r=1}^{t} x_l^r \le 0 \qquad \text{For } t=1,2,\ldots,T \text{ and } b=1,2,\ldots,I,\ldots,I+K$$

or

$$x_b^t - \sum_{r=1}^{t} x_l^r \le 0 \qquad \text{For } t=1,2,\ldots,T,\ b=1,2,\ldots,I,\ldots,I+K \text{ and } l=1,2,\ldots,m$$

Production Tolerance Constraints:

$$L_o\left(\sum_{i=1}^{I+K} TB_i.x_i^{t-1}\right) \le \sum_{i=1}^{I+K} TB_i.x_i^t \le U_o\left(\sum_{i=1}^{I+K} TB_i.x_i^{t-1}\right) \qquad \text{For } t=2,3,\ldots,T$$

Equipment Access and Mobility Constraints:

$$q.x_b^t - \sum_{l=1}^{q} x_l^t \le 0 \qquad \text{For } t=1,2,\ldots,T \text{ and } b=1,2,\ldots,I,\ldots,I+K$$

Table II

Minimum and maximum size of a long-term production scheduling problem

		One constraints per block per period	m constraints per block per period
Minimum size: T=10 and I+K=100000	Number of 0-1 variable Number of constraints	700 000 1 470 069	700,000 7 070 069
Maximum size: T=10 and I+K=1000000	Number of 0-1 variable Number of constraints	10 000 000 21 000 069	10 000 000 101 000 069

where:

P: Unit selling price of the metal

SP: Unit selling cost of the metal

g_i: The average grade of i-th block

R_n: Metal recovery of material if classified in category n

M_{cn}: Unit mining cost of the material, which is classified as the n-th group.

P_{cn}: Unit processing cost of the material which is classified as the n-th group

n: Correct mining category for block i

n': Chosen mining category for block i.

As is clear from the above equation, if the correct mining category for block i is chosen, then the economic loss is zero.

In practice, the ore block grade is not known and it will be represented by a cumulative distribution function (CDF). This method needs realizations of a block grades, which are obtained via the conditional simulation method. In this case the expected economic loss $E[L_n(i)]$ for each scenario n is calculated using the conditional probabilities $(P_n(i)|O))$ and average grades $(\bar{g}_{in}|O)$ for each mining category, generated from O independent realization, i.e.:

$$E\left[L_n(i)\right] = \sum_{j=1}^{N}\left[P_j(i)|O\right]L_j(i) \quad [42]$$

The optimal mining category for block i is that n for which the expected economic loss is minimized, i.e.:

$$L_{opt}(i) = Min\left\{E\left[L_n(i)\right], n = 1,2,...,N\right\} \quad [43]$$

Example

A gold mine sends its mineralized products to the four destinations: waste dump, dump leach, heap leach or mill. The characteristics of these four classes are listed in Table III.

At first one hundred equally probable realizations of the orebody gold grades were generated using Sequential Gaussian Simulation. The results of the simulation for a given block are shown in Table IV.

If the correct mining category for block i to be such that it should be sent to the dump leach (n=2), but incorrectly it is sent to the waste dump ($n' = 1$), then the economic loss due to this misclassification can be calculated as:

$$L_2(i) = \left[(10 - 0.5) \times 0.7 \times 0.45 - 3.5\right] - \left[(10 - 0.5) \times 0.7 \times 0 - 1.5\right] = 1$$

Also for n=3 and $n' = 1$ we have:

$$L_2(i) = \left[(10 - 0.5) \times 1.5 \times 0.7 - 5.7\right] - \left[(10 - 0.5) \times 1.5 \times 0 - 1.5\right] = 5.775$$

The economic loss for the other values of n and n' are shown in Table V.

If the chosen class of block i is n=1, then the expected loss due to its misclassification can be achieved as follows:

$$0 \times 0.35 + 1 \times 0.28 + 5.775 \times 0.2 + 31.612 \times 0.17 = 6.8$$

According to Table V, the optimum expected loss is 1.69; therefore, the optimum class of this block is class 3, it means that it is better to send this block to the heap leach.

In the long-term production scheduling problem, the extraction of each block in each period depends on its economic value in that period. On the other hand, due to changes in price and costs during the time, block economic value varies with time; therefore; it may be possible that the optimum block category in a period be different from the optimum block category in the other periods. Accordingly, the loss function for block i in period t can be expressed as:

$$L_n^t(i) = \left[\left(P^t - SP^t\right) \times \left(\tilde{g}_{in}|O\right) \times R_n - M_{cn}^t - P_{cn}^t\right] - \left[\left(P^t - SP^t\right) \times \left(\tilde{g}_{in}|O\right) \times R_{n'} - M_{cn'}^t - P_{cn'}^t\right] \quad [44]$$

Where P^t, SP^t, M_{cn}^t and P_{cn}^t are the unit metal price, unit metal selling cost, unit mining cost and unit processing cost in period t respectively. In this case the expected economic loss in period t can be calculated using the following equation:

$$E\left[L_n^t(i)\right] = \sum_{j=1}^{N}\left[P_j(i)|O\right]L_j^t(i) \quad [45]$$

Thus, the optimum mining category for block i in period t can be obtained as follows:

$$L_{opt}^t(i) = Min\left\{E\left[L_n^t(i)\right], n = 1,2,...,N\right\} \quad [46]$$

Using this method, the optimum block category in each period is identified.

Conclusion

In this paper we extended a chance-constrained programming with probabilistic parameters and binary integer variables to the open pit long-term production scheduling problems. The model integrates ore grade uncertainty explicitly and economic and mining considerations to generate an optimal life of mine production schedule to meet the required targets with a high level of confidence and low risk. At first, a probabilistic form of the long-term production scheduling model was developed, which contains a stochastic objective function and a series of deterministic and stochastic constraints. The stochastic objective function and also the constraints could not to be handled directly in the optimization process; consequently, using some assumptions a deterministic from of the model was obtained. This process leads to converting stochastic linear equations to deterministic nonlinear equivalents. Meta-heuristics such as the Genetic Algorithm, Tabu search, Simulated Annealing, etc. can be applied in order for this model to be implemented on a large open pit mine. The results obtained from this model are more economical in the long-term than those

Table IV

The results of 100 simulations for a given block

Grade range	0–0.49	0.5–0.99	1–1.99	≥2
Number of realizations	35	28	20	17
Corresponding probability	35%	28%	20%	17%

Table III

The characteristics of each mineralized block class

Mining category	Waste (n=1)	Dump leach (n=2)	Heap leach (n=3)	Mill (n=4)
Grade range (gr/ton)	0–0.49	0.5–-0.99	1–1.99	2
Average grade(gr/ton)	0.4	0.7	1.5	4.5
Metal recovery (%)	0	45	70	95
Selling cost ($ /gr of gold)	0.5	0.5	0.5	0.5
Mining and processing costs ($/ton of ore)	1.5	3.5	5.7	10.5
Gold price ($ /gr of gold)	10			

Table V

The economic loss matrix of the block categorization

		Correct mining category				Expected economic loss
		1	2	3	4	
Selected	1	0	1	5.775	31.612	6.8
mining	2	0.29	0	1.362	14.375	2.81
category	3	1.54	0.538	0	5.887	1.69
	4	5.39	3.68	1.237	0	3.16

obtained from the previous models, because the objective function will force the model to derive the mining sequence through zones where the risk of not achieving the production targets is minimized; therefore, the resultant schedule is feasible in terms of meeting the production targets with a high level of confidence.

The economic loss function method is known as an efficient technique to determine the optimum processing type of the material in each period as it considers ore block grade uncertainty. This can reduce the economic loss due to the misclassification of mineralized block and increase project NPV.

It also considers equipment access and mobility constraints results in generating feasible scheduling in terms of practical excavation of the blocks during the periods they are scheduled. The mining process in this case requires significantly less movement of equipment compared with the schedules produced with traditional optimization formulation. This will improve the achievable project NPV.

References

1. Rovenscroft, P.J. Risk analysis for mine scheduling by conditional simulation. *Trans. Instn Min. Metall.* (Sec.A: Min. Industry),101, May-August 1992. The Institution of Mining and Metallurgy. 1992. pp. A82–A88.
2. Dowd, P.A. Risk assessment in reserve estimation and open pit planning. *Trans. Instn Min. Metall.* (Sec.A: Min. Industry),103, September-November1994. The Institution of Mining and Metallurgy. 1994. pp. A148–A154.
3. Denby, B. and Schofield, D. Inclusion of risk assessment in open pit design and scheduling. *Trans. Instn Min. Metall.* (Sec.A: Min. Industry),104, January—April 1995. The Institution of Mining and Metallurgy. 1995. pp A67–A71.
4. Dimitrakopoulos, R. and Ramazan, S. Managing risk and waste mining in long-term production scheduling of open pit mine, *2003 SME Annual meeting & exhibit* , Feb. 24–26, Cincinnati Ohio. PREPRINT: 03–151. 2003.
5. Godoy, M. and Dimitrakopoulos, R. Managing risk and waste mining in long-term production scheduling of open pit mine, *2003 SME Annual meeting and exhibit*, Feb. 24–26, Cincinnati Ohio. 2003.
6. Ramazan, S. and Dimitrakopoulos, R. Traditional and new MIP models for production scheduling with *in situ* grade variability, *International Journal of Surface Mining,* Reclamation and Environment, vol. 18, no. 2, 2003. pp.85–98.
7. Dimitrakopoulos, R. 1998, Conditional Simulation Algorithms for Modeling Orebody Uncertainty in Open Pit Optimization, *International Journal of Surface Mining, Reclamation and Environment*, vol. 12, 1998. pp. 173–179.
8. Vallee, M. Mineral Resource+ engineering, economic and legal feasibility=Ore reserve, *CIM Bulletin*, vol. 90, 2000. pp. 53–61.
9. Smith, M.L. Integrating conditional simulation and stochastic programming: an application in production scheduling. *ProceedingS of Application of Computers and Operations Research in the Mineral Industry*, 2001, pp. 230–207.
10. David, M.A. *Handbook of applied advanced geostatistical ore reserve estimation*. Netherlands: Elsevier Scientific publisher. 1988.
11. Journel, A.G. and Huijbregts, C. *Mining Geostatistics*. New York: Academic press, 1978. 600 pp.
12. Dowd, P.A. A review of recent developments in geostatstics. *Computers and Geostatistics*, vol. 17, 1992. pp. 1481–500.
13. Dowd, P.A. Risk in minerals projects: analysis, perception and management. *Trans. Instn Min. Metall.* (Sec.A: Min. Industry),106, January—April 1997. The Institution of Mining and Metallurgy. 1997. pp. A9–A18.
14. Lerchs, H. and Grossman, F. Optimum design of open-pit mines. *Transaction CIM*, vol. 58, no. 633, 1965. pp. 47–54.
15. Zhao, H. and Kim, Y.C. A New Optimum Pit Limit Design Algorithm. *23rd International Symposium on the Application of Computers and Operations Research in The Mineral Industries, AIME*, Littleton, Co. 1992. pp. 423–434.
16. Johnson, T.B. and Barnes, J. Application of Maximal Flow Algorithm to Ultimate Pit Design. *Engineering Design: Better Results through Operations Research Methods*. North Holland, 1988. pp. 518–531.
17. Yegulalp, T.M. and Arias, J.A. A Fast Algorithm to Solve Ultimate Pit Limit Problem. *23rd International Symposium on the Application of Computers and Operations Research in The Mineral Industries*, AIME, Littleton, Co. 1992. pp. 391–398.
18. Ramazan, S. and Dimitrakopoulos, R. Production scheduling optimization in a Nickel Laterite deposit: MIP and LP application and infeasibility in the presence of orebody uncertainty. *International Symposium on Mine Planning and Equipment Selection*, 23–25 April 2003, Kalgoorelie, 2003. 5 pp.
19. Ramazan, S. and Dimitrakopoulos, R. Recent applications of operations research in open pit mining, *2003 SME Annual meeting & exhibit*, Feb. 24–26, Cincinnati Ohio. 2003.
20. Gangwar A. Using geostatistical ore block variance in production planning by integer programming. *17th APCOM symposium*, 1982. pp. 443–460.
21. Kall, P. and Wallace, S.W. *Stochastic Programming*, First Edition, John Wiley and Sons, 1994. 305 p.
22. Charnes, A., Cooper, W.W., and Symonds, G.H. Cost horizons and uncertainty equivalents: An approach to stochastic programming of heating oil. *Management Science*, vol. 4, no. 3 April, 1958. pp. 235–263.
23. Charnes, A. and Cooper, W.W. Chance-constrained programming. *Management Science*, vol. 6, 1959. pp. 73–79.
24. Charnes, A. and Coper, W.W. Deterministic equivalents for optimizing and satisficing under chance-constraints. *Operation Research*, vol. 11, 1963. pp. 18–39.
25. Henrion, R. Introduction to chance-constrained programming. http://stoprog.org/SPIntro/INTRO2CCP.HTML. 2005.
26. Richmond, A.J. Maximum profitability with minimum risk and effort. *Proceeding of Application of Computers and Operations Research in the Mineral Industry*, 2001, pp. 45–50. ◆